[AM-03-010] Spatial Interaction

Abstract

Spatial interaction (SI) is a fundamental concept in the GIScience literature, and may be defined in numerous ways. SI often describes the "flow" of individuals, commodities, capital, and information over (geographic) space resulting from a decision process. Alternatively, SI is sometimes used to refer to the influence of spatial proximity of places on the intensity of relations between those places. SI modeling as a separate research endeavor developed out of a need to mathematically model and understand the underlying determinants of these flows/influences. Proponents of SI modeling include economic geographers, regional scientists, and regional planners, as well as climate scientists, physicists, animal ecologists, and even some biophysical/environmental researchers. Originally developed from theories of interacting particles and gravitational forces in physics, SI modeling has developed through a series of refinements in terms of functional form, conceptual representations of distances, as well as a range of analytically rigorous technical improvements.

Keywords: analyzing multidimensional attributes, basic analytical methods, flows and networks, human mobility, modeling, neural networks, spatial process models

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Explanation

1. Definitions

spatial interaction (SI): The aggregate movement of individuals, commodities, capital, or information over (usually geographic) space resulting from a decision process. In some cases, this definition can be extended to include more abstract concepts such as influences among interacting places more generally.

distance-decay: The degree to which distance (be it physical, economic, cultural, environmental, etc.) affects SIs. Generally, the level of SI between two locations is assumed to decrease as the distance between them increases.

calibration: Model calibration, also known as the process of estimation, involves finding the parameters that enable a model, such as a gravity model, to most closely re ect a particular known SI setting. Parameters are also sometimes called coefficients in different literatures.

2. What is spatial interaction?

Spatial interaction (SI) can be defined as the movement of individuals, commodities, capital, and information over (geographic) space resulting from a decision process (Fotheringham & O'Kelly, 1989). SI therefore encompasses research into migration, shopping, recreation, commodity and capital flows, communication, transportation networks, and commuting, as well as animal mobility, spatial dependence (e.g., plants competing for sunlight, water, and soil nutrients), and even some biophysical/environmental phenomena. The fundamental principal underlying these types of interactions is that individuals trade off the benefits of interaction (e.g., travel-to-work, flock migration) with the costs associated with overcoming the spatial separation between them and their possible destination (Fischer, 2001). It is this core concept that has made theories of SI and SI modeling important in terms of understanding spatial behavior across a range of topics. Originally developed from theories of interacting particles and gravitational forces in physics, SI modeling has developed through a series of refinements in terms of functional form, conceptual representations, and a range of analytically rigorous technical improvements.

$$
\mathbf{T} = \begin{array}{c} \begin{bmatrix} T_{11} & \cdots & T_{1j} & \cdots & T_{1m} \\ \vdots & & \vdots & & \vdots \\ T_{i1} & \cdots & T_{ij} & \cdots & T_{in} \\ \vdots & & \vdots & & \vdots \\ T_{n1} & \cdots & T_{nj} & \cdots & T_{nm} \end{bmatrix} \begin{matrix} O_1 \\ \vdots \\ O_i \\ \vdots \\ O_n \end{matrix} \\ \begin{matrix} D_1 & \cdots & D_j & \cdots & D_m & T \end{matrix} \end{array}
$$

Figure 1: Example SI (origin/destination) matrix. The row, column, and total sums (Oi, Dj , and T) represent the origin outflows, destination inflows, and overall level of interaction of the matrix respectively.

The phenomena of interest in most SI research are interactions of "actors" between a set of "origin" and "destination' locations. In nearly all SI analyses, the flows between origins and destinations can be represented using some form of origin/destination (OD) matrix (Figure 1). This type of interaction matrix, **T**, usually represents flows of actors, Tij , between n origins and m destinations. By summing the flows across each row of the interaction matrix, we obtain the observed out flow from each origin, Oi, and similarly, by summing each column of the interaction matrix, we obtain the observed in ow into each destination, Dj . The sum of all flows in the interaction matrix, $T = \sum_{ij} T_{ij}$, represents the overall level of interaction in the OD matrix. Some researchers also conceptualize SI flows as a network or graph (Figure 2).

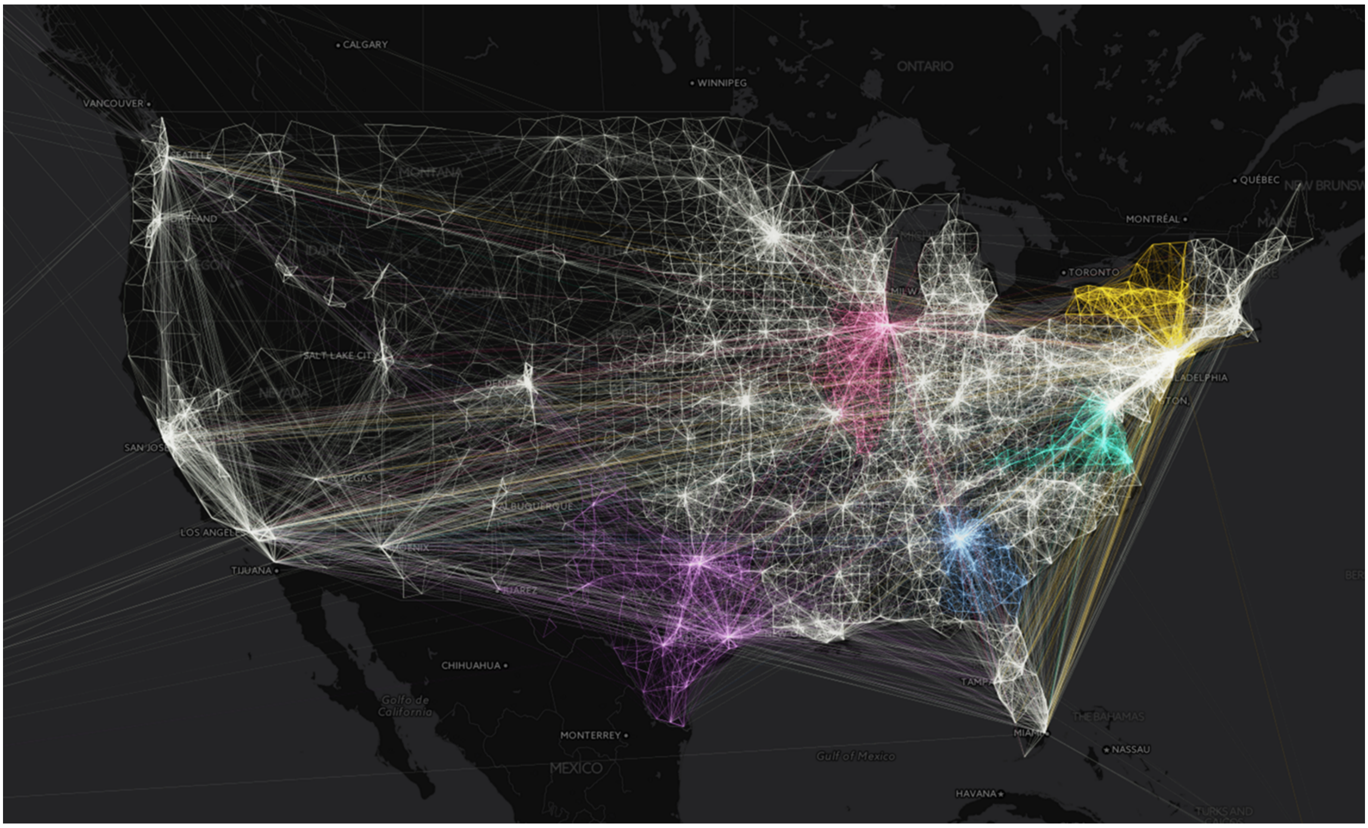

Figure 2. Estimated commuting flows in the (contiguous) United States (U.S.), 2010. This figure represents 138,517,811 flows between counties in the U.S., with flows into Texas, Virginia, Georgia, New York, and Illinois colored for reference. Note the very regional patterns in commuting, and the hub-and-spoke nature of the connections. Clearly, population density and distance play key roles in the propensity to commute between counties in the U.S. Map source: authors. An interactive version of this map is also available online at https://mamataakella.carto.com.

3. A brief history

3.1 Gravity models and social physics

The most well known early attempt to characterize observed regularities in SI data is that of Ravenstein (1885). In his early study, Ravenstein observed that large cities tended to draw migrants from other large cities, and that this effect appeared to drop off with distance (i.e., the number of individuals migrating from one city to another tended to be inversely proportional to the distance between them). This basic observation lead to the development of a mathematical model for predicting migration flows between origins and destinations based on the Newtonian gravity model:

$$T_{ij} = k\frac{P_i P_j}{d_{ij}^2}$$ (Equation 1)

where Tij again denotes the flows between origin i and destination j, Pi and Pj represent the sizes of locations i and j, dij describes the distance between i and j, and k is a scaling parameter used to adjust the magnitude of Tij relative to $PiPj/d_{ij}^2$. Subsequent refinements to this simple model allowed for the specification of Pi and Pj as the populations of i and j (e.g., Stewart, 1941). Realization that the effect of distance and population may in fact vary depending on the context and type of flows being investigated (i.e., the distance decay and population size effect will likely be different for migration than it would be for shopping or travel-to-work) lead some researchers to empirically estimate additional parameters to allow for these variations:

$$T_{ij} = k\frac{P_i^{\alpha} P_j^{\mu}}{d_{ij}^{\beta}}$$ (Equation 2)

where β is a distance decay parameter which represents the degree to which estimated values of Tij decrease with distance, and α and μ are parameters reflecting the relationship between Tij and Pi and Pj, respectively.

Various forms of Equation 2 have been used and refined over the years (e.g., Ravenstein, 1885; Reilly, 1929; Stewart, 1941, 1942; Carrothers, 1956), and while the term gravity model has remained popular, the model itself was widely criticized for its simplicity and overall lack of theoretical grounding. This lead some researchers to develop more theoretical frameworks for the gravity model (Fotheringham et al., 2000).

According to Haynes & Fotheringham (1984), it was not until the work of Huff (1959, 1963, 1964) on consumer behavior and the concept of intervening opportunities of Stouffer (1940, 1960), that truly behavioral interpretations of SI models were formulated. In particular, Huff's probabilistic retail model focused attention on the choice options of the shopper (Roy, 2004), rather than simple competition between retailers. Similarly, Stouffer attempted to explain the movements of migrants based on the number of opportunities at a given destination, keeping in mind that there may be intervening opportunities that might attract a migrant along the way. An additional derivation of the gravity model, this time from economic principals and concepts of utility maximization, was developed by Niedercorn & Bechdolt Jr (1969). However, all these conceptual advancements suffered from several important shortcomings, including the fact that they were attempting to describe aggregate outcomes from individual level behavior.

3.2 Entropy and spatial interaction

The pioneering work of Wilson (1967) produced a family of SI models, which was particularly important because it provided a theoretical justification for SI models. While the justification was based on the statistical mechanics of the model rather than human behavior, it was nevertheless a catalyst for a great deal of additional SI research. Using the language of statistical mechanics, Wilson considered a SI system in which the ows of individuals between origin/destination pairs are simply a "macro state" of the overall system and the individuals moving between origins and destinations are individual "micro states" that combine to produce the observed macro state. The goal then is to choose the macro state that can be constructed from the largest number of micro states, and is, therefore, the most probable. Without additional constraints, the above procedure always yields a solution where all Tij values are as close to equal as possible (i.e., the solution with the maximum entropy/uncertainty).

Wilson recognized that additional information could be used to derive a range of models that could be tasked to solve various SI problems, often resulting in more accurate estimates of Tij . Based on the notation in Figure 1, the type of information that we have on a SI system might include: 1) all outgoing flows of the system Oi, 2) all incoming flows of the system Dj, 3) both Oi and Dj, and 4) the observed total (or average) trip length. Fotheringham and O'Kelly (1989, chap. 2, p. 2-3) provide examples of various situations where different sets of "constraints" on the general entropy maximization SI model might be used. In addition, further control on the model may be gained by incorporating information on the attraction and/or propulsion of origins and destinations via the inclusion of additional variables (attraction factors) in the model. Consequently, different constraint(s) and variables result in different members of the family of SI models.

The simplest of this family of models is the unconstrained SI model, in which there are no constraints on the outgoing and/or incoming flows of the system (which can sometimes lead to relatively poor estimates of Tij). The origin-constrained model, which uses knowledge of the total out flows, is used to reproduce the observed out ows from each origin and allocate them to the various destinations, whereas the destination-constrained model takes as given the in flows to each destination and allocates the interactions to the various origins.

The fourth model in Wilson's family of SI models is the doubly-constrained SI model, which is perhaps most appropriate in situations where prediction is the primary focus, rather than explanation. Since both the out flows from origins and in flows to destinations are built into the model, it reproduces the observed interactions to a high degree, without providing any information on the attractiveness or propulsiveness of the origins and destinations. As a result, doubly-constrained SI models are often used in trip distribution problems.

Extensions to Wilson's entropy models are plentiful, both from a theoretical and practical perspective. Notable theoretical improvements include the Alonso (1978) framework, which provides a general "theory of movement" that includes the "traditional" family of SI models as special cases but is more flexible (O'Kelly, 2004). The Wilson framework has also been extensively applied to real-world geographical problems, though much of this work is conducted within the private sector and therefore remains difficult to find in academic journals (Birkin et al., 2010). Many additional applications are available (e.g., Wilson, 2000; Singleton et al., 2010; Geographical Analysis, 2010, inter alia) and characterize the utility of this modeling framework as an effective decision support tool.

3.3 Local models, spatial structure, and distance decay

SI models have traditionally been applied at a "global" level, meaning that one set of model parameters is generated that applies equally across the entire study region. If the SI system under investigation has spatially varying relationships (i.e., spatial non-stationarity), then the global interpretation will likely be invalid, leading to poor predictions and misleading results.

In order to explore dimensions of spatial non-stationarity and heterogeneity in parameter estimates, "local" SI models, where the flows from only one origin, or to only one destination can be used. These local SI models are generally termed origin- and destination-specific models and can be used to compare model performance, parameter estimates, and behavior across and/or between origins and destinations (Haynes & Fotheringham, 1984).

Origin- and destination-specific local SI models have a rich history in GIScience, especially in the debate over spatial structure and the interpretation of distance decay parameters (Fotheringham, 1981). This debate centered around the observation that in many studies using origin-specific SI models, accessible (i.e., clustered) origins tended to display significantly less negative distance decay parameters than their peripheral counterparts, which lead to differing opinions on the true meaning of the distance decay parameter. Prior to this debate, it was generally assumed that differences in distance decay parameters were the result of variation in spatial behavior and/or preference structures driven by socio-economic conditions (Roy, 2004) and not due to the spatial structure of locations. Based on these assumptions, all else being equal in the SI system, we would not expect any differences in local distance decay amongst locations with varying levels of accessibility. However, many studies reported implausibly wide ranges of local distance decay parameters, with the most accessible locations having less negative parameters, and some accessible locations even taking positive values! Clearly, this is contrary to the assumptions built into gravity-type SI models and it was shown by Griffith and Jones (1980) and Fotheringham and Webber (1980) (among others), that distance decay and the spatial arrangement of locations in the SI system are interdependent (Roy, 2004)

By outlining specifically what it was about spatial structure that was not captured by existing SI models - competition for interactions that each destination faces from all other destinations in the SI system - Fotheringham derived "a new set of spatial-interaction models," termed the competing destinations model (1983a; 1983b; 1984; 1986). This model posits that competition is the result of a multi-stage decision-making process whereby individuals first select a particular milieu (macro destination) and then subsequently select from specific destinations (micro destinations) within this milieu (Roy, 2004). One way of capturing this destination competition is via a Hansen-type (Hansen, 1959) measure of accessibility (though other forms are equally possible). This measure of accessibility is easily integrated into the traditional SI modeling framework to produce, for example, an origin-constrained competing destinations model.

Another recent approach to account for spatial structure in SI models is to add a spatially autocorrelated component into the model. Several techniques have been proposed to do so and motivations typically include flows being potentially dependent upon nearby flows in previous (time) periods or as a remedy for the effects of potentially omitted variables (LeSage & Pace, 2008; Fischer & Griffth, 2008; Chun, 2008).

4. Calibrating spatial interaction models in practice

In the past, it has been common to estimate the parameters of SI models via regression, which involves linearizing the equations by taking the logarithms of each side (Fotheringham & O'Kelly, 1989), so that an equation like (2) becomes:

$$\ln T_{ij} = \ln k + \boldsymbol{\alpha} \ln \mathbf{v}_i + \boldsymbol{\mu} \ln \mathbf{w}_j - \beta \ln d_{ij}$$ (Equation 3)

where **v**i and **w**j may now be interpreted as vectors containing selected origin and destination attributes (rather than just populations Pi and Pj) and α and μ are the corresponding vectors of parameters. This is the well-known log-normal SI model and, while potentially convenient to apply in practice, it suffers from several significant shortcomings, including 1) the unrealistic assumption that flows are (log) normally distributed, 2) bias created by the log-transformation (i.e., back-transformation bias), 3) violation of the assumption of homoscedastic errors, and 4) the problem of zero-valued interactions (i.e., the log of 0 is undefined).

Flowerdew and Aitkin (1982) have argued that many of the above problems stem from an incorrect specification of the model itself and suggest that Poisson regression would be more appropriate. Recognizing that a) the number of agents moving from i to j must be a non-negative integer, b) the population of i is relatively large, c) the movement of agents is independent, and d) assuming that there is a constant probability of an agent in i moving to j, the number of agents traveling from i to j can be considered to be the outcome of a Poisson distributed process with mean λij .

Since λij is unknown, it can be estimated via a SI model, where it is now assumed that λij is logarithmically linked to a linear combination of the logged independent variables in Equation 3, as in:

$$\lambda_{ij} = \exp(k + \boldsymbol{\alpha} \ln \mathbf{v}_i + \boldsymbol{\mu} \ln \mathbf{w}_j - \beta \ln d_{ij}).$$ (Equation 4)

This general formulation has been shown to be equivalent to other forms of SI models, including the entropy maximizing SI models from subsubsection 3.2 (Griffth, 2010). In fact, all four models from Wilson's family of SI models can be operationalized by particular specifications of the explanatory variable vectors in Equation 4. For example, the origin-constrained variant of a simple Poisson SI model is given by:

$$\lambda_{ij} = \exp(k + \varphi_i + \boldsymbol{\mu} \ln \mathbf{w}_j - \beta \ln d_{ij})$$ (Equation 5)

which is essentially an unconstrained model with an origin-specific constant term φ_i (sometimes termed an origin-specific "fixed-effect"). In essence, φ_i represents an origin dummy covariate or set of indicator variables that represent origin-specific effects. These location-specific effects can also be estimated using the Furness algorithm (Sen & Smith, 1995; Griffith & Fischer, 2013).

Furthermore, since Poisson regression models belong to the family of generalized linear models (GLM), Poisson SI models benefit from several key advantages, including 1) standard diagnostics for model specification, 2) tests for assessing the significance of explanatory variables, 3) efficient algorithms for calibration, and 4) the potential for extending the model to include additional explanatory variables. For an outline of the Poisson SI modeling framework and some recent software for calibrating them, the reader is referred to Oshan (2016).

5.0 Non-parametric spatial interaction

A recent trend in SI modeling is the use of non-parametric techniques, which means either that no parameters need to be estimated or there are no underlying distributional assumptions, or both. The primary focus of these models is predicting SI and includes neural network SI models and "universal models."

5.1 Neural Spatial Interaction

Neural networks (NN) are a computing framework that draw on an analogy to neurons in the brain (Miller, 2009). Given sufficient input data and an appropriate NN structure, it is possible to approximate or "learn" any data-generating process with few a priori assumptions about the data. Openshaw (1993) was the first to propose the use of NNs to model SI flows, which are often noisy, non-linear, and may vary from place to place. Consequently, NN SI models have been used in various contexts, such as predicting telecommunication traffic, journey-to-work trips, and commodity flows (Fischer, 2002; Mozolin et al., 2000) and generally boast higher predictive capabilities than legacy SI models.

There are generally three steps involved in building a NN SI model. First, a feed-forward NN architecture must be specified. For SI models, this has generally consisted of a two-layer system (Openshaw, 1993; Mozolin et al., 2000; Fischer, 2013). The second step focuses on deriving the values for the network weights that connect the layers, also called network "training." Input data are first split into training, validation, and testing sets. Then training is carried out by adopting a loss function, which helps assess the ability of a set of weights derived using the training data to predict the validation data. By minimizing the loss function, it is possible to obtain optimal weight values. In order to find a level of model complexity that is generalizable and, therefore, not overt to the training data, the third step involves training networks with different topologies (i.e., the number of weights) and evaluating their effectiveness by choosing the model that produces the lowest prediction error rate on the test dataset. More in-depth methods, such as "early stopping" or "regularization" may also be used to achieve a compromise between low error rates and generalizability (Fischer, 2013).

Several studies have compared the predictive capabilities of NN SI models to legacy SI models. While Openshaw (1993), Fischer & Gopal (1994), and others have reported increased accuracy using NN SI models, some have found that in certain circumstances, the legacy SI models outperform NN models, possibly due to NN models overfitting to the training data, thereby leading to poor "out of sample" predictions.

5.2 "Universal" Models

The so-called universal models take their name from the fact that their proponents suggest that they can be applied to different types of SI phenomena (e.g., migration, commuting, animal movements) or in various study regions using a constant set of underlying assumptions. One of the earliest universal models is the so-called "Radiation" model (Simini et al., 2012), which earns its title from an analogy to radiation and absorption processes that yields an equation that does not include any parameters.

The radiation model contrasts gravity-type models in that it does not explicitly consider distance in the computation of T_{ij} . It only uses the distance between an origin and a candidate destination as a radius within which alternative destinations may be found. The sum of the attractiveness for all alternative destinations is then interpreted as the deterrent of interaction compared against the attractiveness at the candidate destination. This interpretation makes the radiation model similar to Stouffer's intervening opportunities model (Stouffer, 1940, 1960). Several efforts have been made to compare the predictive capabilities of the radiation model to a variety of other SI models (Simini et al., 2012; Masucci et al., 2012; Lenormand et al., 2016) with two important conclusions: 1) the performance of the radiation model varies over different scales, implying the radiation model is not truly universal, and 2) when a comparison between the radiation model and its proper gravity-type model counterpart is made, the gravity model performs better, likely due to the flexibility provided by its estimated parameters.

While the radiation model has perhaps received the most attention, other universal models have also been developed. The population-weighted opportunities model (PWO) was designed as a variant of the radiation model specifically for finer scale intra-urban flows (Yan et al., 2013). By recognizing the relatively higher mobility of populations within cities, it assumes flows are a function of the attractiveness of a candidate destination compared to the attractiveness of all other destinations, striking a resemblance to the competing destinations model. A universal model of commuting networks has also been proposed (Lenormand et al., 2012), which is inspired by the previously introduced doubly-constrained gravity model.

In contrast to NN SI models, which require data-intensive model training, universal models carry out prediction of SI without the use of any prior data, which may be useful in data-scarce scenarios. However, neither universal models nor NN SI models permit inference and therefore do not provide a framework for model building, hypothesis testing, or regional comparisons. That is, they do not facilitate the investigation of the spatial processes that generate SI, which is a major limitation.

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