[CV-04-014] Terrain Representation

Abstract

Terrain representation is the manner by which elevation data are visualized. Data are typically stored as 2.5D grid representations, including digital elevation models (DEMs) in raster format and triangulated irregular networks (TINs). These models facilitate terrain representations such as contours, shaded relief, spot heights, and hypsometric tints, as well as automate calculations of surface derivatives such as slope, aspect, and curvature. 3D effects have viewing directions perpendicular (plan), parallel (profile), or panoramic (oblique view) to the elevation's vertical datum plane. Recent research has focused on automating, stylizing, and enhancing terrain representations. From the user's perspective, representations of elevation are measurable or provide a 3D visual effect, with much overlap between the two. The ones a user can measure or derive include contours, hypsometric tinting, slope, aspect, and curvature. Other representations focus on 3D effect and may include aesthetic considerations, such as hachures, relief shading, physiographic maps, block diagrams, rock drawings, and scree patterns. Relief shading creates the 3D effect using the surface normal and illumination vectors with the Lambertian assumption. Non-plan profile or panoramic views are often enhanced by vertical exaggeration. Cartographers combine techniques to mimic or create mapping styles, such as the Swiss-style.

Keywords: aspect, contours, elevation, hachures, hillshading, map design techniques, profile, relief shading, slope

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Explanation

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1. Definitions

2.5D grid representation: A regular or irregular x,y grid in which each grid cell or vertex has one and only one z-value.

3D effect: A representation in which the viewer is given a visual impression of variations in the x,y,z values.

aspect: The compass direction in which the terrain's z-value changes most rapidly in the downslope direction.

block diagram: An oblique view of terrain that also includes cutaway views of the subsurface, often along the edges of the map's extent.

contour: A line connecting points of equal elevation.

curvature: The second derivative of an elevation surface.

digital elevation model (DEM): A data model used to process, store, analyze, and display elevation data.

elevation: The height or depth of a terrain element with respect to a vertical datum plane such as sea level.

hachure: Short lines drawn in the direction of steepest slope and often varying in weight to create relief shading.

hillshading: (See relief shading)

hypsometric tinting: The application of colors to terrain based on elevation values above or below a vertical datum plane such as sea level.

illumination vector: A theoretical direction from which terrain is lighted for 3D effect. It is used to calculate relief shading.

Lambertian assumptions: Terrain is a diffuse reflector and the brightness of surface elements is the dot product of the illumination and the surface normal vector.

oblique view: (See panoramic view)

panoramic view: A viewing direction of a terrain representation that is neither perpendicular nor parallel to the x,y vertical datum plane.

physical model: A scaled-down real-world 3D display of terrain.

physiographic map: A plan or oblique view that represents a map extent while sketching landforms and often includes relief shading.

plan view: A map viewed perpendicular to the x,y vertical datum plane. Maps in plan view often rely on relief shading to create a 3D effect.

profile view: A terrain representation viewed parallel to the x,y vertical datum plane. Profile views typically represent one trace across the x,y plane.

relief: Changes in elevation.

relief shading: Variations in shades of grey, or dark and light contrasts, that provide a 3D effect by assigning gray-scale values predicted with the Lambertian assumptions.

rock drawing: The depiction of rock outcrops or bedrock exposures, typically on a large-scale map of mountainous terrain.

scree patterns: The depiction of loose rocks covering a slope, typically on a large-scale map of mountainous terrain.

slope: A measure of the deviation from horizontal of a terrain element.

surface normal vector: A vector perpendicular to each surface element of the terrain used to calculate relief shading.

Swiss-style: A style that combines various methods of terrain representation such as relief shading, contouring, rock drawing, and scree patterns in a manner consistent with the long-established practices of Swiss cartographers that creates a three-dimensional relief impression.

vertical exaggeration: Scale factor applied to elevation values.

2. Introduction and Data Models

Representing terrain on maps has historically been a cartographic objective of special attention (Harvey 1980), and continues to offer interesting challenges and creative opportunities to modern-day cartographers. Terrain representations begin with elevation values measured from some vertical datum plane such as sea level. While limited numbers of spot heights and soundings were historically recorded directly on topographic and bathymetric maps respectively (Imhof, 1982), voluminous modern elevation data require effective data models.

Elevation data are typically stored in **2.5D grid representations** (Weibel & Heller, 1991), in which each grid element or its vertex has a unique elevation value. 2.5D grids generally provide an adequate representation of the terrain, as the land's surface is continuous and smoothly varying, and data collection platforms operate well above the terrain surface. The representation can be a raster grid composed of typically square grid cells with one elevation value per cell, or a triangulated irregular network (TIN) in which the three vertices of each triangle have a unique elevation value. Both are referred to as **digital elevation models (DEMs)**.

3. Viewing Direction

Viewing direction is an important concern of all terrain representations. Views are typically from some direction above the terrain and thus could be imagined as originating from any point on a large hemisphere surrounding a much smaller, centrally placed terrain model. Looking down on the terrain from the hemisphere's apex results in a **plan view** (Robinson

et al., 1995, Slocum et al. 2009), while looking across from any point on the lower edge of the hemisphere provides a **profile view** (Imhof, 1982). These views are oriented perpendicular and parallel to the elevation values' vertical datum plane respectively, which approximates a horizontal plane for large-scale maps. Views from all remaining directions are **panoramic** or **oblique views**.

Profiles typically are drawn as a line of varying z-value to show changes in elevation in the view along an x,y trace on a map. Such profiles typically do not attempt to represent foreground or background details. Profiles commonly use **vertical exaggeration** to stretch displays in the z-direction to highlight visually subtle variations in **relief**, or changes in elevation. Profile views that also show details of the subsurface are referred to as geologic cross sections.

Panoramic views display the terrain surface from an oblique perspective. These views show changes in the surface's x, y, and z values across the view's extent, providing an intuitive representation of the terrain, but also resulting in hidden areas as landforms in the foreground block views of the background. Panoramic views also use various shading techniques discussed below to enhance the 3D effect, and often use vertical exaggeration to highlight relief.



Figure 1. A panoramic view of Mt. McKinley (Denali) hand-painted by Heinrich Berann for the US National Parks Service. Patterson (2000) discusses his analysis and automation of Berann's methods.

The panoramic maps of Heinrich Berann are striking examples of such views (Figure 1)**.** Patterson (2000) documents how Berann's perspective views used a curved surface to

foreshorten the distance to mountaintops and vertically exaggerate the terrain. The physiographic maps of Raisz (1931) (Figure 2) and Harrison (1941) are two examples of plan or panoramic base maps that sketch panoramic views of landforms for 3D effect. Visvalingam and Dowson (1998) automated panoramic mapping to create similar, sketched effects. Jenny and Patterson (2007) developed the plan oblique relief method, which shows landforms from their side view on a plan basemap. Panoramic views that also show cutaway views of the subsurface along the map's edges are referred to as geologic block diagrams, and Lobeck (1924) offers quantitative techniques for their construction.

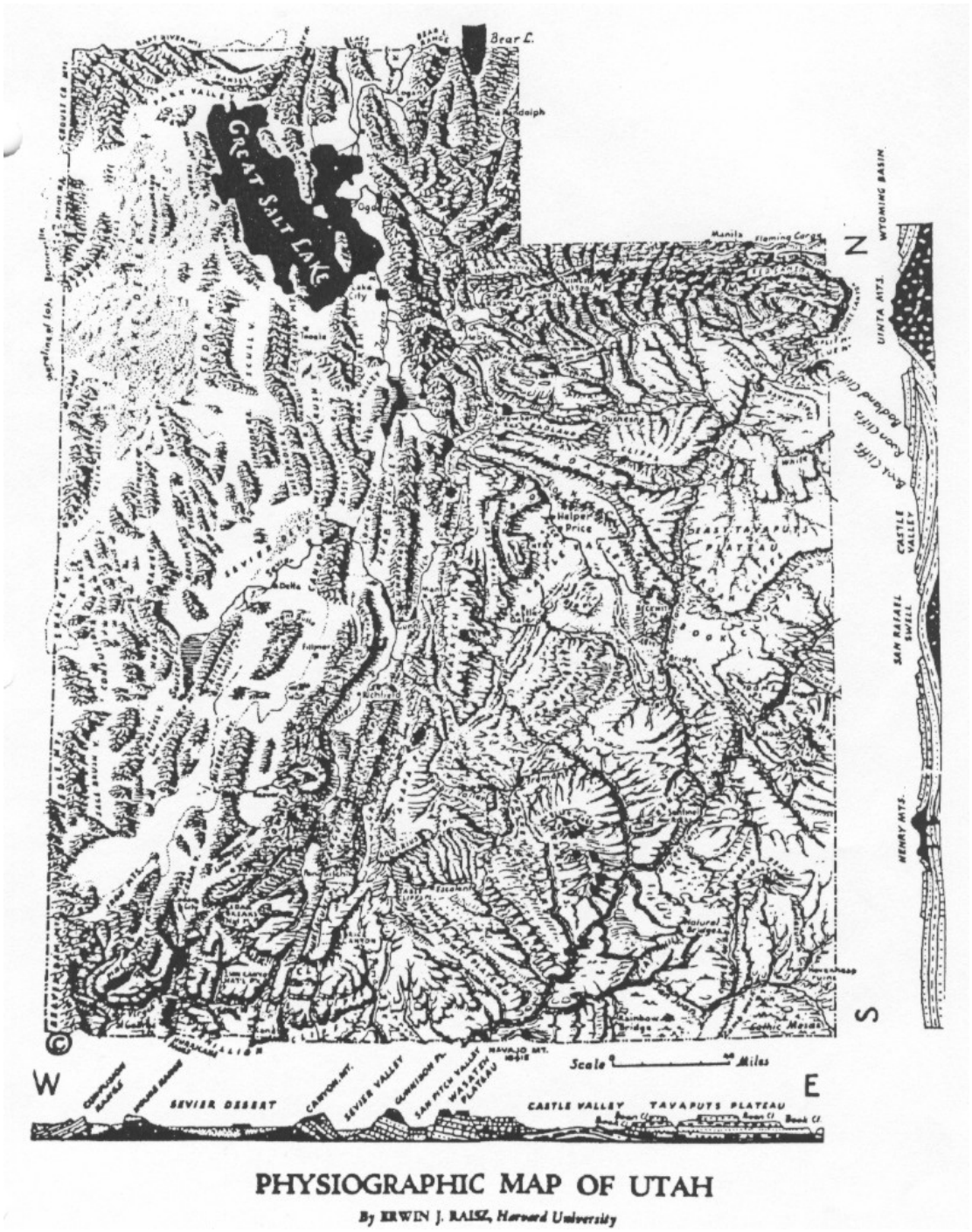


Figure 2. A physiographic map of Utah hand-drawn by Erwin Raisz. Although the basemap is a plan view, the sketches of landforms show variations in the x, y, and z directions as well as relief shading. Raisz also included two geologic cross sections. Reprinted with permission from John Wiley and Sons, Inc.

Plan views offer special challenges in terrain representation (Slocum, 2009), as the entire view is designed to show changes in the x,y directions, with no intrinsic technique to show the **3D effect**. As a result, a number of methods have been devised to represent terrain in plan view. These techniques can be broadly categorized into two categories: 1) those interested in a measurable representation of elevation values, and 2) those interested in creating a 3D effect. The following sections will explore techniques in these categories.

4. Terrain Representations in Plan View - Measurable

Elevation values can be measured from plan views, making terrain representations useful for map reading, interpretation, and analysis (see Map Reading, Map Interpretation, Map Analysis). Early representations include **contours**, isarithmic lines – a common thematic map type – that connect points of equal elevation (Imhof, 1982, Robinson et al., 1995; see Common Thematic Map Types). Measuring or approximating differences between contours allows for an assessment of the steepness of the terrain. The use of an appropriate contour interval (e.g., 20 feet) and index contours (e.g., every 5th contour of 100 feet drawn with a thicker line) are important design considerations. These criteria will change with map scale and the steepness of the terrain, resulting in the need for generalizing contours.

Hypsometric tints assign classified or unclassified color schemes to elevation across the map's extent (Imhof, 1982). These colors can represent variations seen with elevation change in a particular area (Imhof, 1982, Jenny & Hurni, 2006, Leonowicz et al., 2009). Imhof (1982) includes a number of hypsometric color schemes for elevation and bathymetry in Plate 5. Patterson and Jenny (2011) account for global changes in the colors of natural environments of world regions with their cross-blended hypsometric color schemes (Figure 3). Hypsometric color schemes can also be designed with hue and saturation, with relief shading modulating the lightness of the colors (White, 1985).

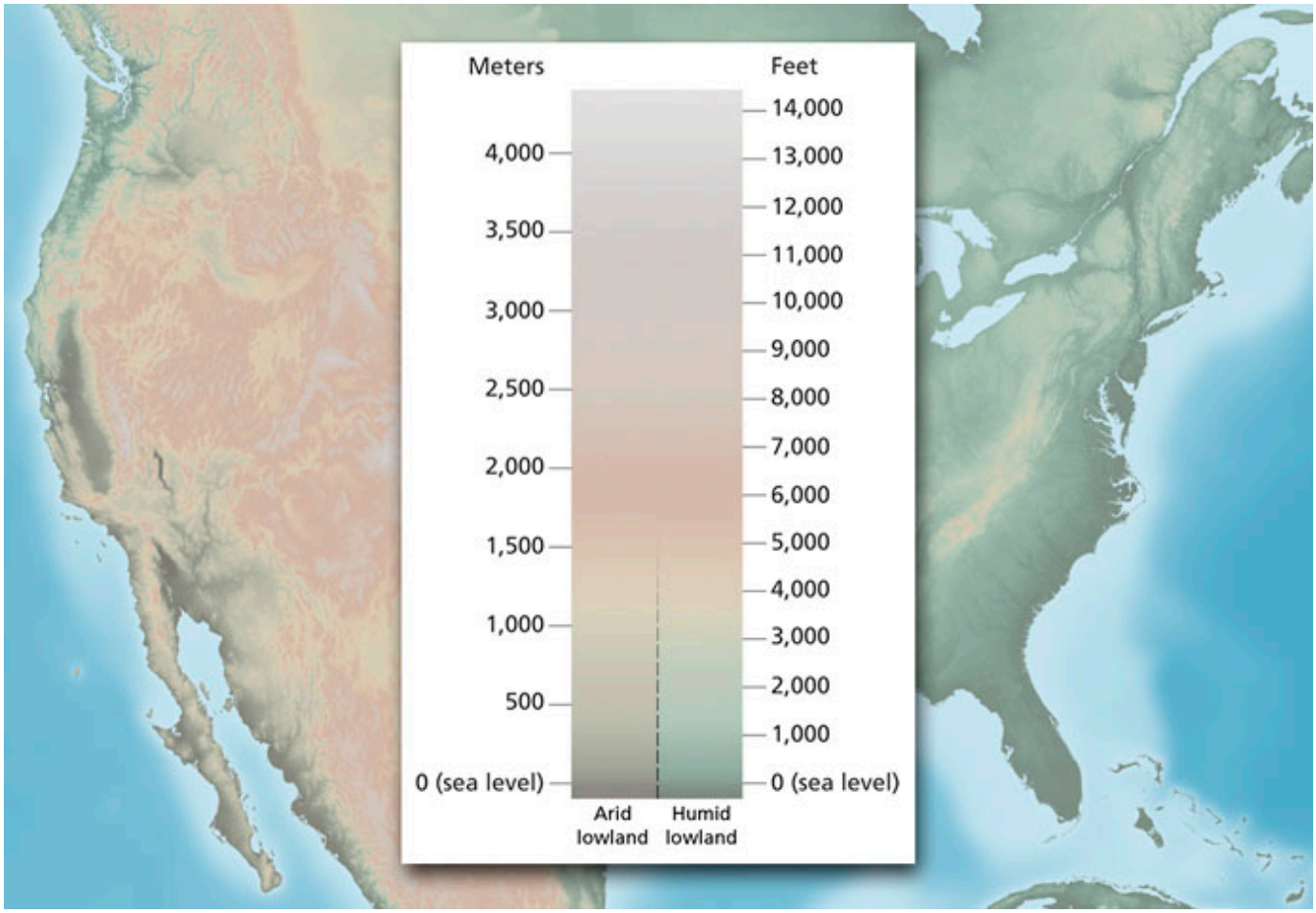


Figure 3. A cross-blended hypsometric tint map of the conterminous United States with a different color scheme for arid and humid lowlands (from http://www.shadedrelief.com/physical/pages/about.html). Reprinted with permission from Tom Patterson.

These techniques allow users to approximate elevation values or make direct measurements, and in some cases to calculate derivative functions. For example, given two elevation values with consistent changes in elevation between them, the user calculates **slope** from the distance and difference in elevation. Similarly, the **aspect** vectors can be constructed normal to contour lines, pointing downslope. These calculations can also be made on gridded data across the terrain (Chang, 2013), resulting in classified or unclassified slope and aspect maps.

One consideration of all of these representations is that they do not offer a strong 3D effect to most map users. For example, closed contours representing a hill will show the plan shape of the hill (e.g., circular versus oval), and the spacing of the contours will provide a shading effect (e.g., black contours on a white background approximate shades of gray). This shading, however, is one described by Imhof (1982) as “the steeper the darker,” and is generally considered to result in a poor 3D effect. As a result, terrain representation has focused on a number of methods that combine the 3D effect with interesting, artistic designs.

5. Terrain Representations in Plan View - 3D Effect

A focus of terrain representation research is the creation of a 3D effect with shading and sometimes a casting of shadows. The approach necessitates illuminating the terrain from a theoretical light source originating form the surface of a hemisphere surrounding the terrain, such as the viewing hemisphere discussed in Section 3. A single **illumination vector** would originate at the hemisphere and cast parallel rays across the terrain. An illumination vector from directly overhead creates a vertical shading effect, and vectors from the base of the hemisphere would cast shadows over much of the terrain model. Illumination vectors from the northwest and at an intermediate inclination between horizontal and vertical seem to provide a preferable 3D effect (Imhof, 1982).

The dot product of the illumination and a **surface normal vector** to each surface element assumes a diffuse Lambertian reflector and is calculated as the cosine of the angle between the two vectors in 3D space (Wiechel, 1878). The result in a map showing shades of gray that are proportional to the amount of light received on each terrain element given its orientation to the direction of illumination. This **relief shading** technique is similar to the artistic effect of chiaroscuro and creates a strong 3D effect (Horn, 1982). Like slope or aspect, **relief** or **hill shading** values can be read directly from a shaded relief map, but only provide the shade of gray predicted by the Lambertian assumption.

Imhof (1982) includes interesting examples from the historical development of relief shading in the first chapter of his book, and dedicates his ninth chapter to shading and shadows cast under oblique lighting. The latter represents the culmination of hand-shading in the **Swiss-style** (Figure 4), including adding color highlights to illuminated and non-illuminated terrain. Seasoned cartographers began with a contour framework, and applied shading, taking into consideration the orientation of terrain elements with respect to a theoretical illumination source. They often varied the direction of illumination across the map’s extent to enhance terrain structure and highlight important terrain features. Horn (1982) published a review article that paid homage to hand-drawn relief shading but also reviewed seminal work in automating such techniques, including research conducted by Yoeli (1959, 1965, 1966, 1967), Brassel (1974), Brassel et al. (1974), and Peucker et al. (1974).

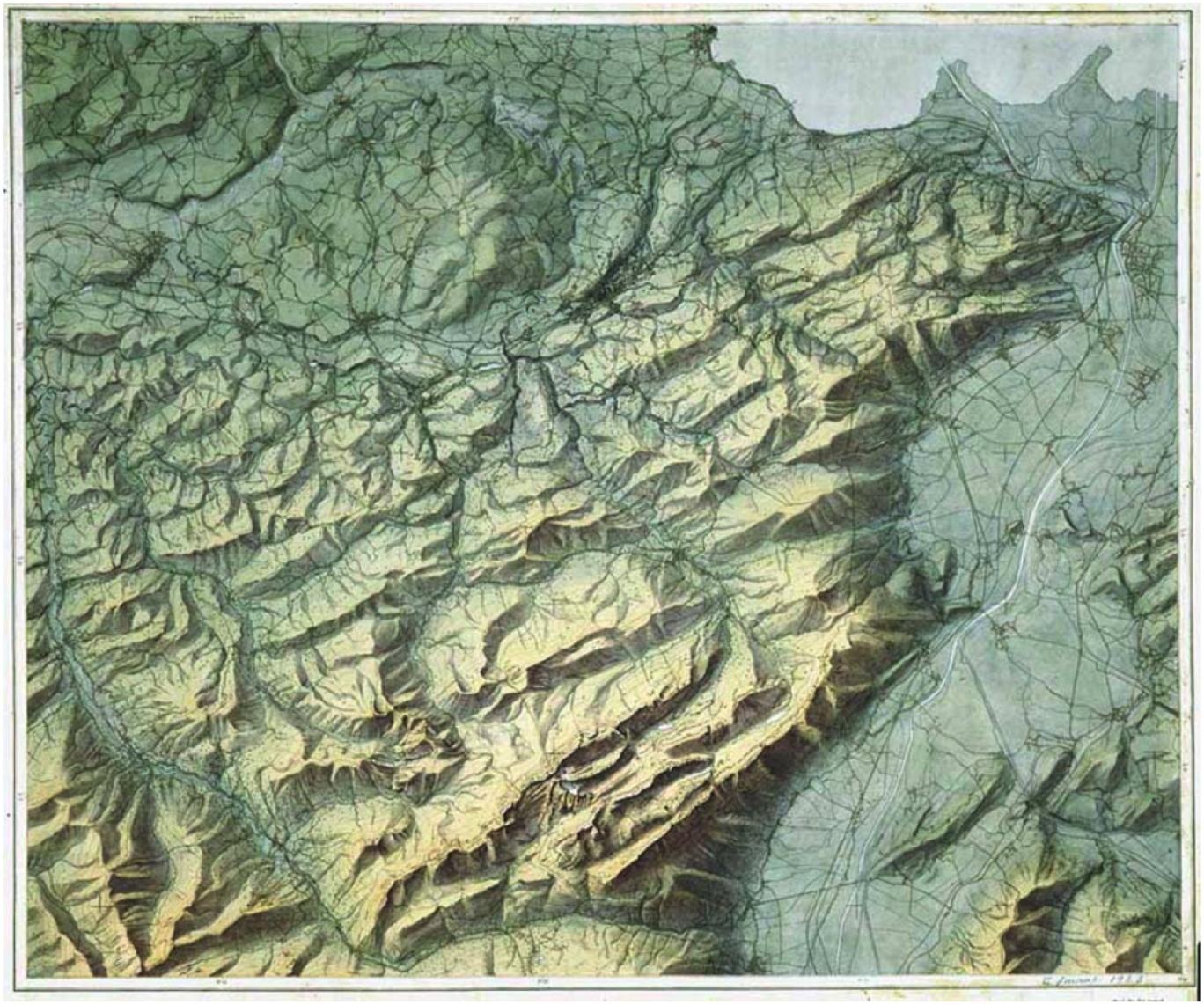


Figure 4. A relief shaded map of the Swiss Appenzell area with hypsometric tinting and color highlights on illuminated and non-illuminated terrain, hand-rendered by Imhof (1923). Reprinted with permission from Swisstopo and the Imhof family.

Two watershed events in automating relief shading occurred in the United States near the end of the 1980's. The first was an iconic shaded relief map of the conterminous US created by Pike and Thelin (1989). The second occurred in 1990 when Esri added a major subsystem (GRID) for raster processing to ARC/INFO 6.X that included its "hillshading" command. Adding relief shading to maps became part of many cartographic workflows, and researchers explored options for adapting the default technique and its assumptions to create enhanced or stylized relief shadings. A portion of Pike and Thelin's map and examples of enhanced relief shading methods are included in Table 1.

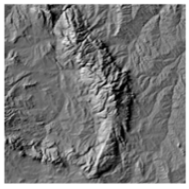
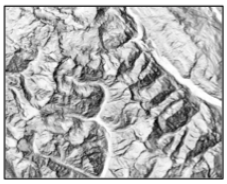
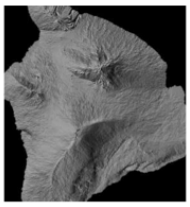
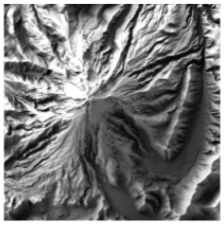
Table 1: Relief Shading Methods			
Method	**Example**	**Description**	**Suggested uses**
Classic relief shading		A portion of the relief shaded map of the United States covering the Bighorn Mountains, Wyoming. (Pike and Thelin 1989)	Default relief shading found in numerous GIS.
Local Illumination Variations		Relief shading using two light sources and adjusting the inclination of the illumination direction according to slope. (Veronesi and Hurni 2015)	Use to enhance ridges or valleys parallel to the general aspect direction of illumination. Use to highlight rock outcrops or emphasize the transition from valleys to slopes.
Oblique and Vertical Illumination		Relief shading combining oblique illumination from the northwest with vertical illumination. (Patterson and Hermann 2004)	Use to highlight features associated with steep slopes, such as landslides or avalanches.
Multiple Illumination Directions		Relief shading of the island of Hawaii using four weighted illumination directions with azimuths of 225°, 270°, 315°, and 360°. (Mark 1992)	Use to enhance ridges or valleys parallel to the general aspect direction of illumination.
Sky Model Illumination		Relief shading of Mt. Hood, Oregon using 250 illumination sources to approximate a turbid sky model. (Kennelly and Stewart 2014)	Use to add soft shadows to terrain maps and to enhance details on portions of the terrain facing away from the illumination direction.

Table 1. Examples of relief shading methods. The classic method used by Pike and Thelin (1989) has been enhanced by adding additional illumination sources from various directions, varying the intensity of the illumination, and locally altering the direction of illumination. Images reprinted and/or modified with permission from the following authors: Tom Patterson, Fabio Veronesi, Lorenz Hurni, Patrick Kennelly and James Stewart.

One line of research focused on locally varying the orientation of the illumination vector to highlight specific landforms. Imhof (1982) makes a strong case for the importance of such adjustments, and Brassel (1974) proposes such automated changes in selected areas. Jenny (2001) first devised an interactive approach to locally adapt the lighting direction that especially benefited landforms parallel to the primary illumination direction. More recent research by Varonesi and Hurni (2015) used two illumination directions and corrects tone by elevation and slope, while an innovative method by Martsen and Jenny (2015) adjusts

illumination directions using ridgelines and valley lines derived from a DEM.

An alternative line of research proposed combining multiple shaded relief layers for an enhanced relief effect. Imhof (1982) suggested combining oblique shading with vertical, and Patterson and Hermann 2004 define an automated method. Other researchers have used multiple illumination vectors, such as the red-green-blue light sources of Hobbs (1999), while Mark (1992) and Patterson (1997) illuminate terrain from different directions, selecting and processing the results. Numerous illumination vectors can approximate the light distribution of a surrounding hemisphere. Yokohama et al. (2002) use a measure of openness to show how much brightness from a uniformly lit sky would reach each terrain element. Kennelly and Stewart (2006, 2014) adapt existing sky models that define the distribution of luminance in the sky to shade and cast shadows on terrain (Figure 5).

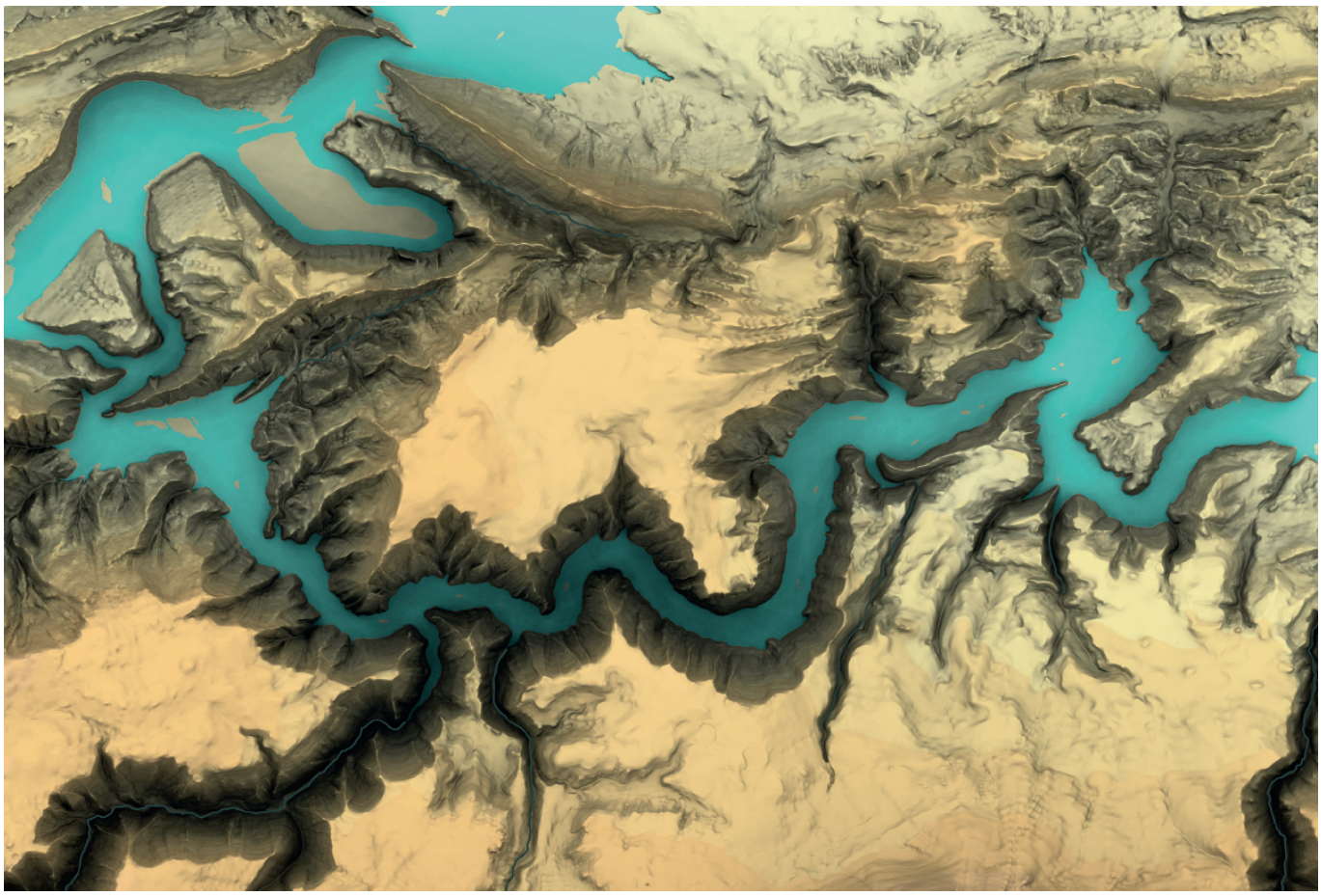


Figure 5. A map of the Flaming Gorge reservoir using hypsometric tints and shaded with an overcast sky model using the techniques described in Kennelly and Stewart (2014). Reprinted from Kennelly and Stewart (2015) with permission.

6. Stylizing Relief Shading

Certain stylized relief shading techniques focus on creating or enhancing the 3D effect (Table 2). Contours typically give a vertical illumination effect, but Tanaka (1950) derived a method for creating oblique lighting with **illuminated contours**. Beginning with a medium gray background, contours on the illuminated sides of terrain are drawn in white and those on the non-illuminated sides are drawn in black, with thickness varying by the dot product

of the illumination and aspect vectors. Researchers later automated his method, including Peucker et al. (1974), Etyon (1984), Kennelly and Kimerling (2001), and Eynard and Jenny (2016).

Table 2: Stylized Relief Shading Methods

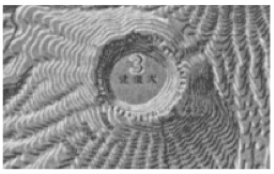
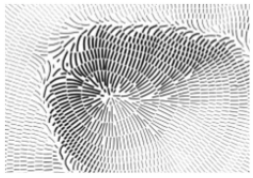
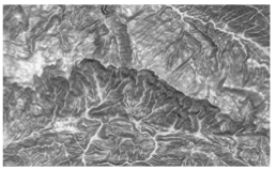
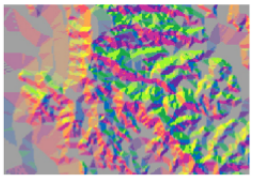
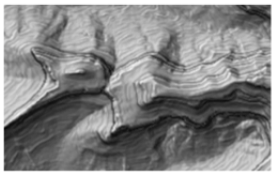
Method	Example	Description	Suggested uses
Illuminated contours		Contours vary in thickness and in black/white based on orientation from illumination source (Tanaka 1950)	Combine with area hypsometric tints between contours, or use dark and light hypsometric colors for illuminated contours.
Inclined contours		Traces resulting from the inter-section of inclined planes with terrain, drawn thicker on non-illuminated surfaces (Robinson and Thrower 1957)	Combine with relief shading to enhance the 3D effect. Use in selected areas to mimic physiographic representations of landforms.
Hachures		Short lines drawn in the direction of steepest slope using surface flowlines (Samsonov 2014)	Combine with hypsometric tints or symbolize hachures with an aspect color scheme. Use in selected areas to highlight steep slopes or landforms.
Slope		Slope shading attenuated to the map user's perceptual bias of overestimating slope (Pingel and Clarke 2014)	Combine with hypsometric tints. Use for representing terrain that has natural vegetation, buildings, etc.
Aspect		Aspect directions represented by hues of varying lightness with chroma varying with slope (Brewer and Marlow 1993)	Use for aspect-specific mapping, such as solar potential. Use color scheme for lines such as hachures.
Curvature		Profile curvature highlighting geologic units of the Grand Canyon (Kennelly 2008)	Combine with relief shading and hypsometric tints to add texture to the 3D effect. Use profile and plan curvature to highlight cliffs and drainages respectively.

Table 2. Examples of stylized relief shading methods that create or enhance the 3D effect, with a brief description and suggestions for their uses. Images reprinted with permission of John Wiley and Sons, Inc., the British Cartographic Society, CaGIS, and Elsevier; and modified with permission from Pingel and Clarke.

Tanaka (1932) also devised **inclined contours**, traces made by intersecting evenly spaced parallel planes oriented between horizontal and vertical with the topography. Robinson and Thrower (1957) enhanced the technique, and the method was automated by Peucker et al. (1974), Yoeli (1976), and Kennelly (2002). Kennelly (2015) reviews its design

considerations, as the inclined traces are not optimal for 3D effect.

Hachures are short lines drawn in the slope direction (Lehmann 1799). Hachure maps using narrower and thicker lines on illuminated and non-illuminated terrain respectively give a 3D effect. Imhof (1982) cites the Swiss Dufour map series as achieving its apex of artistry. Yoeli (1985), Kennelly and Kimerling (2000), and Samsonov (2014) developed automated methods, although this technique proves difficult to fully automate.

Slope, aspect, and curvature maps have been successfully adapted to create a 3D effect. Pingel and Clarke (2014) begin with slope shading, but attenuate it to the perceptual bias by which users overestimate slope. Moellering and Kimerling (1990) use aspect and slope to provide a relief shading effect, with the luminance of aspect colors defining a modified cosine curve. Brewer and Marlow (1993) modified this technique by using hues that maximize lightness differences for aspect and decreasing the lightness of chroma with increasing slope. Kennelly and Kimerling (2004) varied the luminosity of aspect colors to complement variations in value of relief shading. Kennelly (2008) also combines curvature with relief shading in a manner consistent with sky illumination models.

7. Combining Techniques

Cartographic displays of terrain representation often combine multiple techniques in an aesthetically pleasing manner. Imhof (1982) dedicates chapter 14 of his book to the “interplay of elements” such as hypsometric tints, relief shading, slope shading, contours, rock drawings and hachures. Considerations include the scale of the map display, its intended use, and the nature of the symbology (e.g., area versus line symbols).

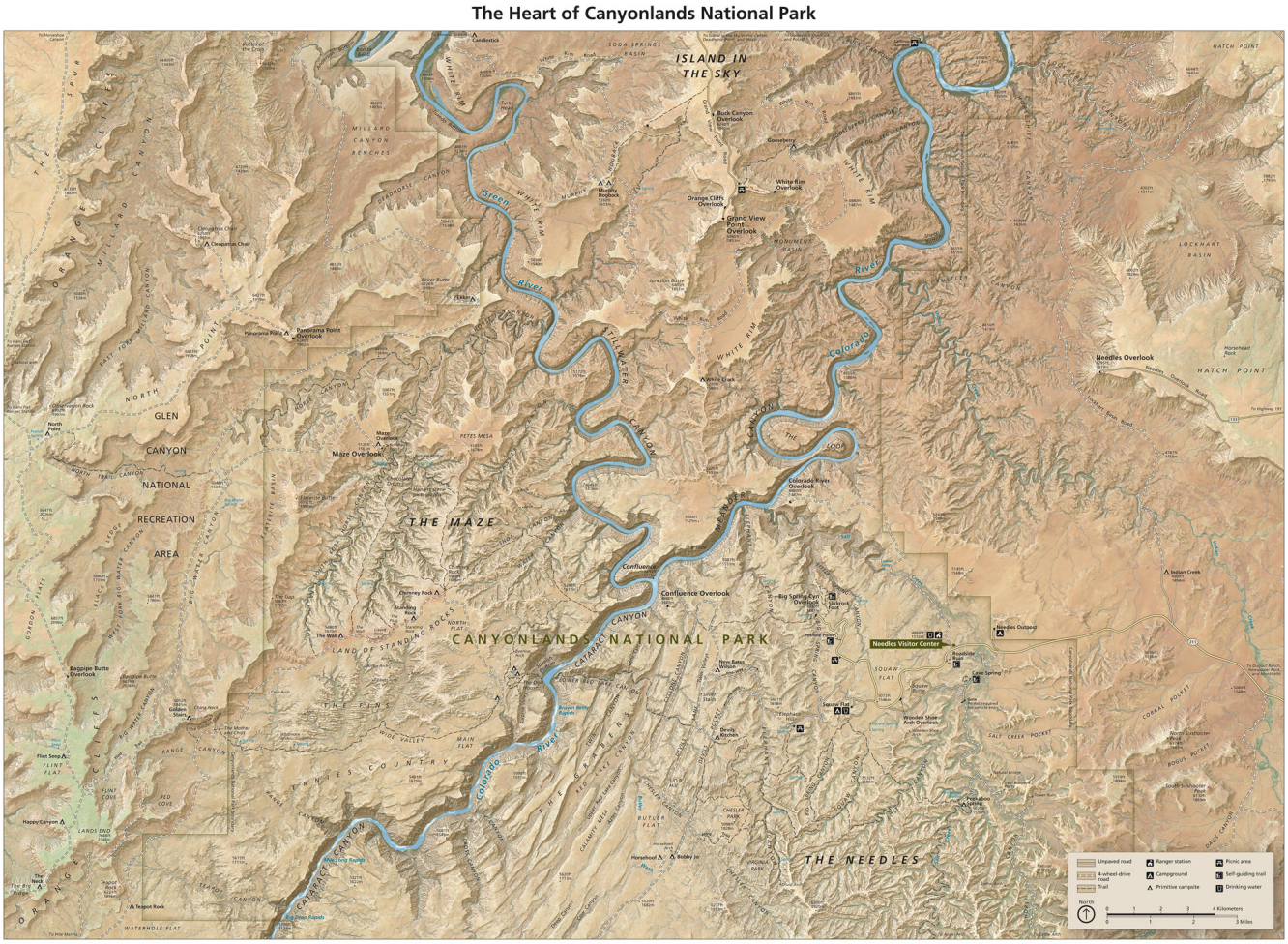


Figure 6. Patterson (2017) uses relief shading, texture shading and layer tints derived from aerial photographs to create “The Heart of the Canyonlands.”

Patterson’s (2017) map “The Heart of Canyonlands National Park” is one exemplar in combining techniques (Figure 6). It achieves a stunning visual effect by combining of relief shading, texture shading using a fractional Laplacian operator (Brown 2010), and aerial photographs as a tinting layer.

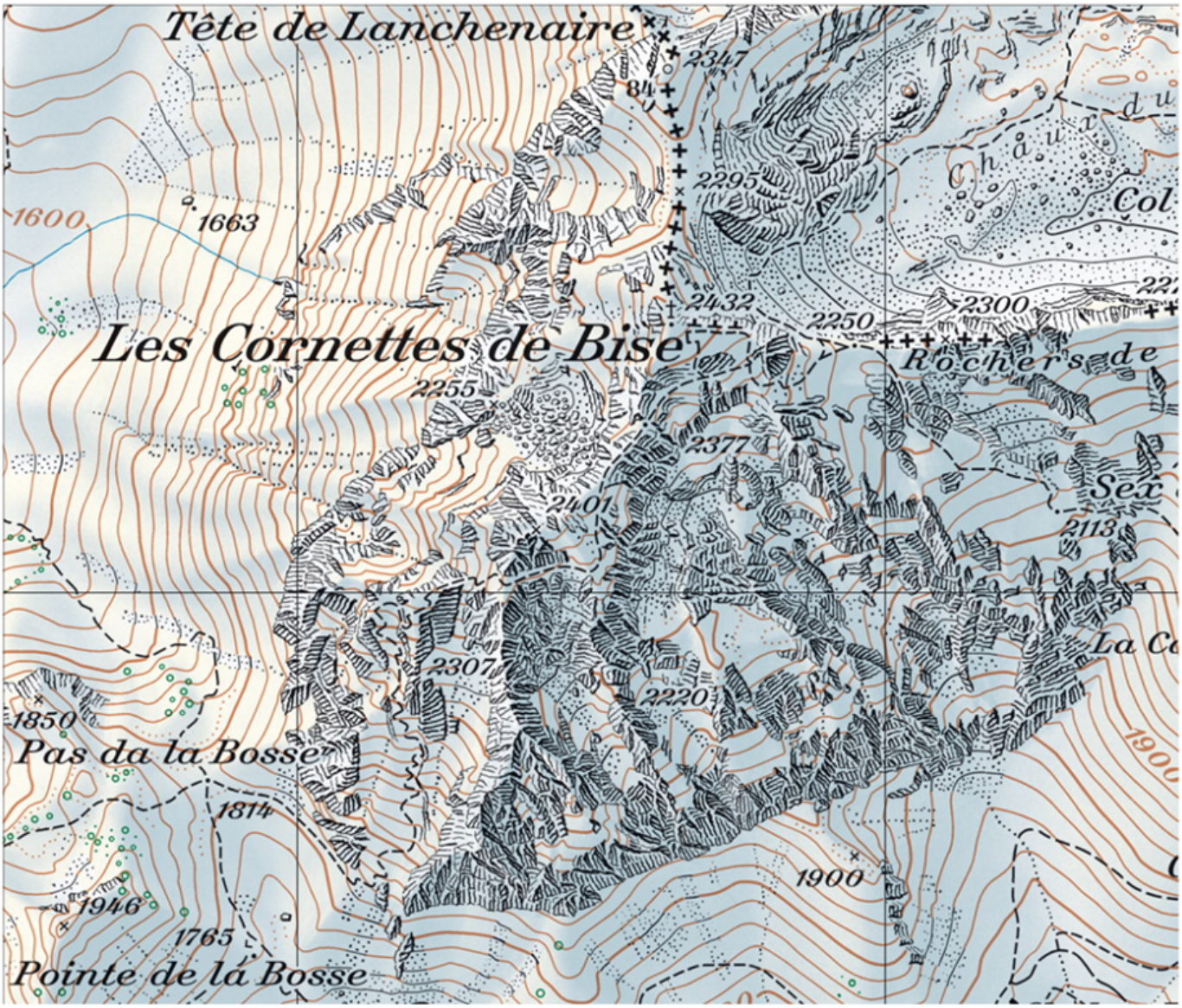


Figure 7. A map with computer-automated Swiss-style relief shading, rock drawings, and scree patterns (Jenny et al. 2014). Reprinted with permission from the British Cartographic Society.

In terrain representation research, a great deal of effort has gone into automating the Swiss-style of terrain mapping. This includes relief shading and hypsometric tinting (Jenny & Hurni, 2006), and special localized enhancements such as **rock drawings** (Gilgen & Jenny, 2010; Jenny et al., 2014) of rugged outcrops (Figure 7), and **scree patterns** (Jenny et al., 2010) in areas of loose rocks.

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