[DM-02-009] The Hexagonal Model

Abstract

Hexagons offer significant advantages in geospatial analysis due to their ability to model relationships efficiently while maintaining isotropy, minimizing edge effects, and improving computational efficiency. While circles are ideal for proximity-based modeling, they do not tessellate. Only three geometric shapes—rectangles, triangles, and hexagons—can fully cover a plane without gaps. Among these, hexagons provide superior data storage, visualization, and analysis benefits. However, hexagons have been underutilized due to historical inertia in geographical information systems (GIS), which favor grid-based Cartesian representations. Additionally, the absence of a standardized global hexagonal framework has hindered data comparison and integration. With increasing computational power and growing global-scale research needs, hexagons are gaining traction in geospatial analysis. Their inherent geometric strengths make them a compelling alternative for modeling spatial relationships, and their adoption is expected to rise significantly within the geospatial community.

Keywords: tessellation data models

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Explanation

1. Overview
2. History
3. Isotropy
4. Edge Effects
5. Modeling Neighborhoods and Paths
6. Computation
7. Discrete Global Grid Systems

1. Overview

One of the core tenets of geospatial analysis is understanding the relationship of features to their neighbors. Tobler's First Law of Geography states that 'near things are more related than distant things' and the geometric shape that is most suitable for modeling relationships, based on centrality and proximity, is the circle. Yet, the circle does not allow for a tessellation, or repeating pattern without gaps or spaces over a surface.

There are only 3 basic geometric shapes that can be used to completely cover the surface of a plane with no gaps or overlaps (defined as a tessellation). These geometric shapes are the rectangle, triangle, and hexagon. When comparing hexagons to these other geometric shapes for the purposes of representing raster data or aggregating vector data, there are a

number of unique advantages for data storage, visualization, and analysis.

The key areas where hexagons exhibit strong advantages over the other tessellating geometric shapes include:

- Isotropy
- Modeling neighborhoods and paths
- Edge effects
- Computational efficiencies

Each of these advantages will be discussed and covered in detail throughout the paper.

Yet, the broad use of the hexagon for data representation has been hampered for a few key reasons. First and foremost is sheer inertia owing to the computational data storage and visualization tools used in geographical information systems, which strongly reinforced data storage and analysis to grid-based approaches. This in turn grew out of the ‘familiarity’ and ‘ease’ of representing geographic features in a map-centric cartesian grid.

Secondly, the lack of a scalable, global hexagonal standard hampered the reproducibility of research as well as the ability to compare and combine datasets. With the rapid growth in computing power and data storage, research questions posited at global scales, as well as the inherent geometric benefits produced by hexagonal forms in analyzing, storing, and representing spatial phenomena, hexagons will continue to see rapid growth in their use in the geospatial community.

2. History

One of the earliest known examples of the use of hexagons to represent geographical concepts was in the work of Walter Christhaller. In his thesis ‘Central Places in Southern Germany’, Christhaller worked to build a theoretical model to describe social systems. Christhaller was trying to understand and discover laws that shape the distribution and size of towns and settlements (Christhaller, 1966).

In much of Christhaller’s work, there is mention of the ‘natural’ order of centralization. The notion of a town square, or a church, forming the ‘central’ place in a community from which settlements grow.

One of the other defining features of Christhaller’s work is the notion of ‘central places of a higher order’, which is visible in the diagram as the ‘higher level’ hexagon that incorporates overlapping and tessellating hexagons at larger scales.

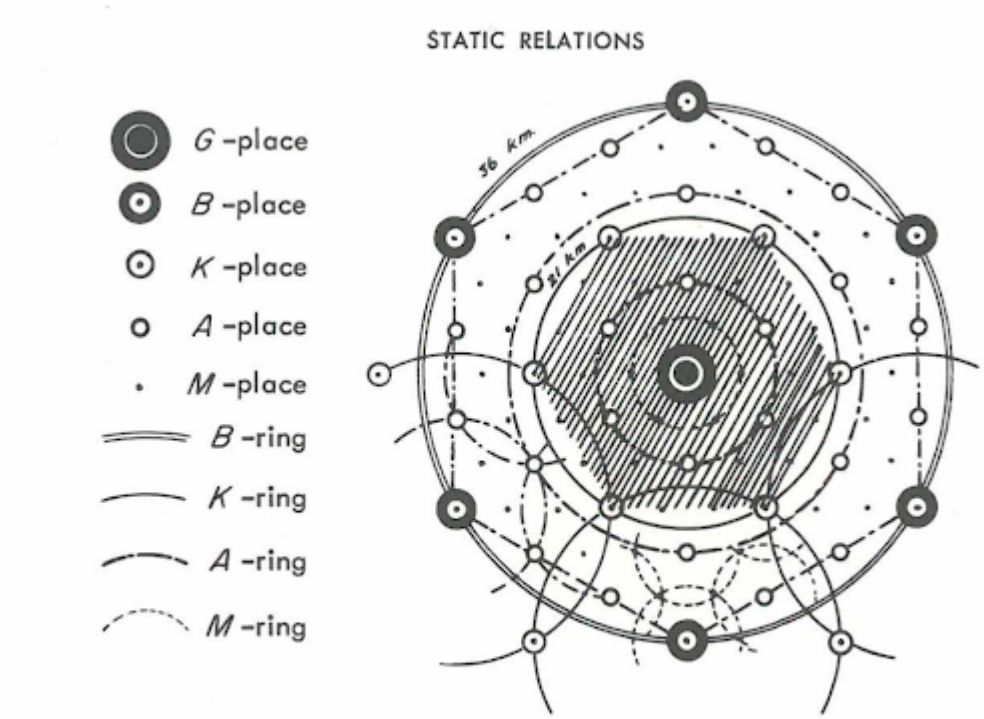


FIGURE 1

A System of Central Places According to the Market Principle

Figure 1. A System of Central Places According to the Market Principle (Christhaller, 1966).

While Christhaller's work is one of the most well-known uses of hexagons for spatial representation, it does not stand alone. There are also many examples of hexagons being employed for the use of population estimates (Barnes, J. 1940; Robinson, 1961) and market and trade areas (Losch, 1938; Getis, 1962), to name a few, prior to the widespread adoption of geographic information systems (GIS). GIS is mentioned as a demarcation, in the context of the wide-spread adoption of grid-based methods for storing and visualization information which it abetted.

As Goodchild (2018) notes, technical limitations at the inception of GIS, led to the 'map', or 2-dimensional cartesian representation of space, driving the methods and tools of GIS, vs. a global representation. Also, the 'grid' was simply one of the most familiar visual representations for users. It was best suited to data storage, printing, and graphical displays (Sahr, 2003).

As noted by Birch (2007), hexagonal tessellations have been widely used for military maps and geospatial simulations, yet grid based approaches for modeling and analyzing data has been the preference of the majority of researchers, for many of the reasons noted above.

While a wide-range of research and work had been conducted on the benefits of hexagonal models for storage and analysis, it was not until the early 90's when computing gains and the wide prevalence of global datasets and global scale research, led to the growth in research and development of discrete global grid systems, of which, hexagonal models play an ever increasing role, owing to many of the benefits of hexagons that have already been discussed in detail.

3. Isotropy

The hexagon exhibits the most isotropy of any tessellating geometric shape. Each vertex of the hexagon is equidistant from its central point, unlike a rectangle, where the central point of each edge is closer than the vertices from the central point.

Owing to the isotropic nature of overlapping hexagons, this allows for Euclidean distances to be modeled with the least amount of distortion. The variability of straight-line distance to measured distance is the shortest in a hexagonal model. Thus, hexagons offer a better approximation of cartesian distance (Luczak & Rosenfeld, 1976).

The diagram below highlights how, as straight-line distance increases, the lattice distance in a hexagonal model has the smallest ratio (Birch, 2007).

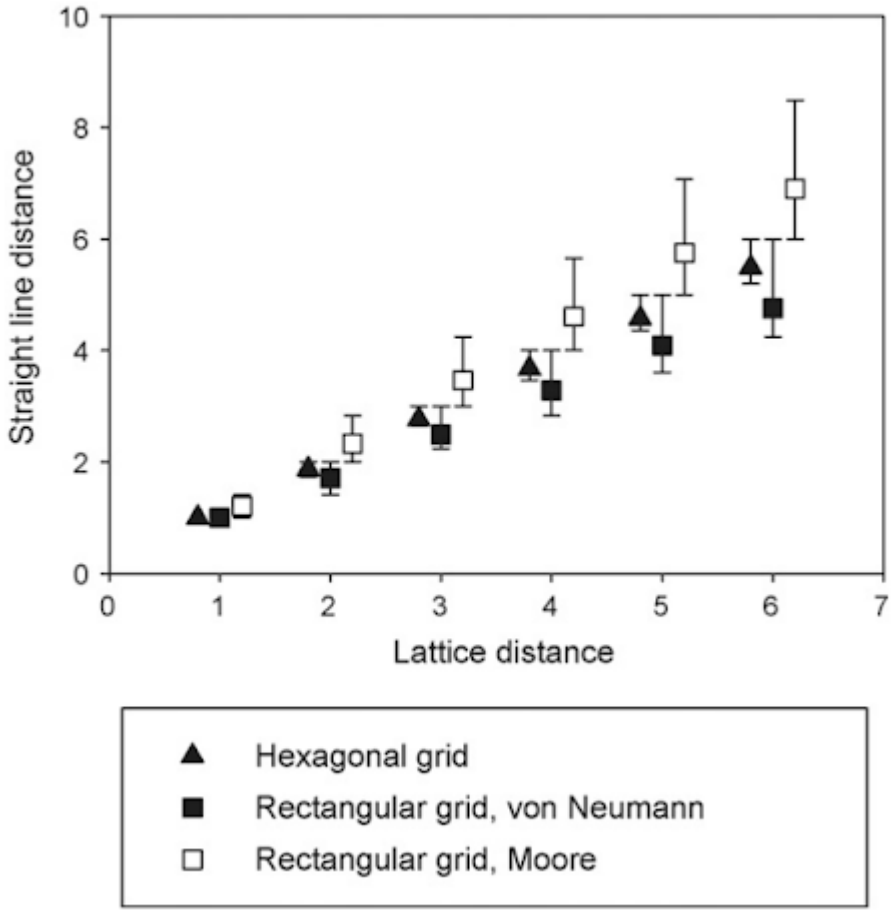


Figure 2. A comparison of straight line distances compared to lattice distance (Birch, 2007).

Isotropy is closely related to measures of compactness. Compactness is a measure of the distance between all parts of a shape, the circle having the smallest measure. Owing to its shape, hexagon offers the best measure of compactness versus triangles and rectangles.

4. Edge Effects

When observations or records are summarized in a rectangular grid or cell, features or pixels that fall in the ‘corners’ are counted or weighted just as evenly as those near the center point.

In the diagram below, each of the yellow circles are the same size. While there is obviously some overage with the hexagon, it is minimal compared to the square. The overage found on the square from the squares central point is significantly greater, which highlights how pixels and features counted or included in square cells are not as closely related to the central point.



Figure 3. Highlighting the edge effects of a hexagon vs. a square. Source: author.

When measuring the distance from the center of the feature to the edge, the distance has the least amount of variation in a hexagon. In a square, the distance from the center point to the corners is more pronounced. By utilizing hexagons, populations are less likely to find themselves added to the wrong population groups.

5. Modeling Neighborhoods and Paths

All maps, as well as data stored and visualized via geographic information systems, are representations of reality. If the goal of an analysis or visualization is to model the relationships between observations on a planar surface, the methods employed should seek to mostly closely approximate their ‘real-world’ relationships. When modeling neighborhoods and paths, this can be accomplished more accurately using hexagons vs. triangles or rectangles.

A challenge when using rectangular shapes to model neighborhoods and paths, is that the relationship between features must be defined, as shown in the diagram below.

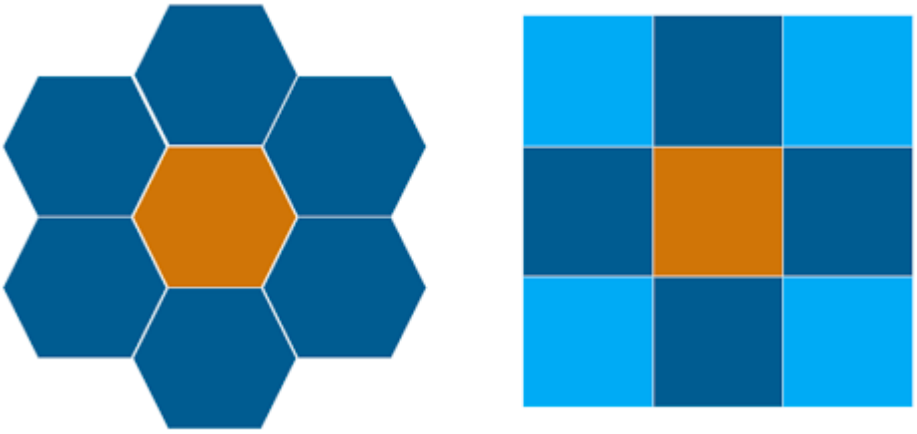


Figure 4. Modelling neighbors with hexagons and rectangles. Source: author.

For the central rectangle, do the diagonal neighbors have equal weighting? Should the diagonal neighbors not be counted at all because they do not share an edge with the central feature? These considerations do not come into account with a hexagonal model as all 1st order neighbors exhibit adjacency (Birch, 2007).

Unlike rectangles, all of a hexagon's neighbors have symmetrical equivalence (Birch, 2007). The distance from the midpoint of a hexagon to the midpoint of any of the 6 neighbors is equal. The nearest neighbors are therefore all equivalent, unlike the relationship that exists between a rectangle and its nearest neighbors. This allows for a simpler representation, and movements between neighbors can be more accurately modeled (Ke, 2019).

6. Computation

A growing area of research has looked at the benefits of hexagonal data models in large data environments, namely for machine learning. As the volume and quantity of data continues to grow exponentially, hexagons can provide needed computational advantages. Specifically, with high-performance computing - there is evidence to show that hexagons provide for the most efficient means of processing and representing spatial information (Sahr, 2012). For example, hexagon rasters are 13.4% more efficient at sampling circularly bandlimited signals (Petersen & Middleton, 1962), and processing algorithms on hexagon rasters are 25-50% more efficient (Mersereau, 1979). Staunton (1989) implemented a set of edge detection operators on a hexagonal raster and realized over 40% better performance compared to equivalent operators on square grids (Sahr, 2011).

As computing continues to advance, barriers that were once in place to inhibit the use of hexagons or other data storage models will continue to evaporate. As datasets become larger and any small computational gains compound, hexagonal data models have the potential to provide for analytical performance gains that should overcome any of the drawbacks. As discussed in depth by Goodchild (2018), GIS is no longer confronted with the same technical limitations owing to computing storage and processing as it was at its outset.

While outside the bounds of this article, related research has recognized hexagons as one of the most efficient resource packing geometries in the natural world, namely in

honeycombs, soap bubbles, human photoreceptors, dragonfly eyes, among others (Roorda, 2001). It's fascinating to note that all of our visual stimuli is being interpreted via hexagonal geometric structures.

7. Discrete Global Grid Systems

A Discrete Global Grid (DGG) is simply a partitioning of the earth's surface. One of the most common and long-standing DGGs is the latitude/longitude geographic coordinate system. Based on the geographic coordinate system one employs, coordinates can be assigned to locations on a sphere based on a grid and a level of precision implied (Sahr, 2003).
The geographic coordinate system, however, has limitations when it comes to analyzing and representing data at a global scale. As cartographers have grappled with for centuries, distortions in area, shape, distance, etc. are inherent with any planar projection.

Advancements in computing and data capture have provided for research at a global scale to expand rapidly, with that expansion, the need for systems better suited for storing, visualizing, and analyzing this information also arose.

For an in-depth discussion of history and development of DGGs, please review the work of Kimerling (1999) and Sahr (2003). These papers both review the properties of multiple DGG schema of various tessellating geometric shapes. As noted in the papers, the concern for most researchers is locating a 'best fit' model that provides equal-area cells and a nested hierarchy.

One of the most widely used "hexagonal" specifications, H3, was developed by Uber and released under an open-source Apache 2 license. H3 is a hierarchical geospatial index that uses a hexagonal tiling scheme. Each of the hexagons in the H3 specification has its own unique index, providing users with the ability to conduct global or local analyses or create visualizations.

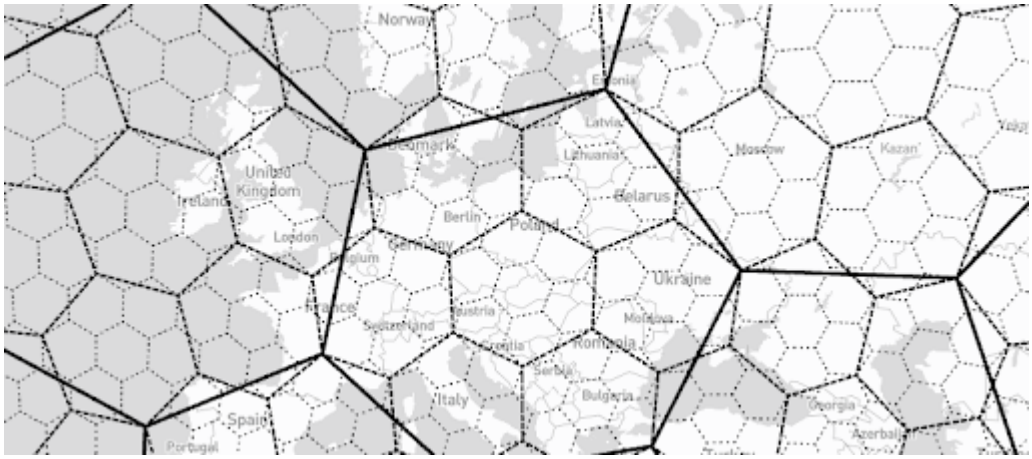


Figure 5. Screenshot of the H3 tiling scheme over Europe. Source: https://h3geo.org/.

There have been many other research and commercial activities that have resulted in DGGs. Just as is the case with map projections, there is no 'best DGG'. Different systems exist and continue to be developed that are the best fit based on user needs. One recent example from the literature is the EASE-DGGS, a DGG built specifically for earth observation data (Thompson, 2022).

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