[DM-02-041] Fuzzy Models

Abstract

Fuzzy logic and fuzzy set theory provide the conceptual basis for modeling the spatial continuity of many geographic entities lacking clear boundaries and a practical framework for managing uncertainty and imprecision in GIS models. The cognitive basis of fuzzy models is rooted in prototype category theory, which posits that humans perceive and interpret the world through categorical structures. Categories, formed through human interaction with the environment, exhibit non-uniform membership, with certain members serving as better exemplars, known as 'prototype effects'. When classifying geographic features that display prototype effects, fuzzy logic provides an effective method for their representation and for modeling the associated uncertainty in the classification process. One of the primary applications of fuzzy models in GIS is fuzzy inference for decision-making. A fuzzy inference system consists of a knowledge base comprising fuzzy membership functions and a set of fuzzy rules. Common applications include predictive soil mapping, risk modeling, suitability analysis, and site selection using multi-criteria decision analysis. Recent advancements in fuzzy GIS models leverage hybrid models that integrate fuzzy multi-criteria decision analysis with advanced analytics such as AI/machine learning.

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Explanation

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1. Introduction

Fuzzy logic extends Boolean logic based on the mathematical theory of fuzzy sets (Zadeh, 1965). While classical sets employ binary memberships, where an element either entirely belongs or does not belong, fuzzy sets allow for gradual or partial membership. An element can belong to a fuzzy set to varying degrees, from complete membership (membership degree of 1) to no membership at all (membership degree of 0), with intermediate degrees in between. The degree of membership is represented by a membership function, assigning a value between 0 and 1 to each element based on its conformity to the criteria of the fuzzy set. Notably, if all membership values except 0 and 1 are removed, the fuzzy set reduces to a crisp set. Fuzzy set theory, therefore, is a generalization of the classical set theory.

The practical significance of fuzzy logic lies in its ability to model human reasoning, which often deals with subjective and ambiguous information. By recognizing, representing, and interpreting information that are vague and lack certainty, fuzzy logic can capture and process this uncertainty more effectively than conventional methods (Klir and Yuan, 1995).

This has led to its extensive applications in modeling and inference systems where human-like decision-making processes are desired. Fuzzy models employ linguistic variables to represent human knowledge in natural language terms (e.g., "high," "low") and use fuzzy rules to define relationship among these variables for inference, effectively mimicking human decision-making.

Conceptually, fuzzy logic and probability address different forms of uncertainty. Probability quantifies the frequency or likelihood of events or conditions occurring making it suitable for modeling stochastic uncertainty. In contrast, fuzzy logic addresses the degree to which an observation falls within a vaguely defined set, commonly employed to capture imprecision stemming from semantic vagueness. Despite their inherent differences, the application of fuzzy logic and probability concepts has become intertwined across various areas such as classification, pattern recognition, and uncertainty modeling. Because the theory of probability, whether objective or subjective, offers a rigorous framework and robust tools for modeling random phenomena, it has often been applied to model inherently fuzzy concepts. It should be noted, though, that while probabilistic methods can approximate membership functions, they do not alter the fundamental nature of fuzziness inherent in the concept under study.

In summary, fuzzy logic and fuzzy set theory provide both the conceptual basis for representing entities lacking clear boundaries and a practical framework for managing uncertainty and imprecision in diverse fields such as control systems, artificial intelligence, decision-making, and pattern recognition. Fuzzy logic systems are often domain specific since defining appropriate fuzzy sets and membership functions need to be tailored to the application field.

2. Fuzzy Representation of Spatial Objects

2.1 Cognitive Basis

One of the central concerns of cognitive psychology is the process of categorization and classification (Rosch, 1978). Humans perceive and interpret the world through the use of categories. The act of classification—the ability to distinguish that A is different from B—is possible because of categories. Classical category theory posits that the world consists of natural partitions, and categorization serves to assign objects to appropriate partitions (Hahn and Randamscar, 2001). Under this view, all instances of a category share common features that are singly necessary and jointly sufficient for defining the category (Smith and Medin 1981). Consequently, an instance possessing all defining features qualifies for full membership in the category, whereas an instance lacking any defining feature must be completely excluded from the category.

The classical view of categories came under intense criticism in the 1970s, leading to the emergence of new theories (Smith and Medin, 1981). These new theories viewed categories as conceptual structures formed through human perception and interaction with the environment (Hahn and Ramscar, 2001). Categorization involves comparing a new instance to previously established (but highly malleable) mental representations of the category, while classification hinges on the similarity of the instance to these representations. Among the newer theories, prototype theory (Rosch, 1978) gained widespread acceptance. Prototype theory highlights that category membership is not uniform, with some members serving as better exemplars known as "prototype effects" (Lakoff, 1987). The prototype of a category is an average or composite representation of actual instances, reflecting central tendencies in their properties. It is more similar to some members of the category than others and is itself an abstract construct, not necessarily an instance (Smith and Medin, 1981).

Many geographic features, such as soil, vegetation, and land cover, are conventionally categorized into discrete classes and mapped as polygons on area-class maps (Mark and Csillag, 1989). Classification is often based on some properties that change continously across both the geographic and feature spaces, resulting in classes with internal heterogeneity and ambiguous boundaries. For instance, in the mapping of soil polygons based on soil-landscape units (Hudson, 1992), definitions are inherently relative when referring to convex versus concave positions or steep versus gentle slopes. The resulting soil polygons exhibit evident prototype effects, where certain locations are more typical instances of a soil type than others. Consequently, fuzzy models have emerged as effective tools for modeling the spatial continuity of these geographic entities and capturing their inherent prototype effects.

2.2 The Similarity Model

The vector data model relies on crisp boundaries and is therefore ineffective in representing geographic phenomena where boundaries between classes are ambiguous or where transitional zones exist. The traditional raster data model is not effective in handling prototype effects either as it assigns discrete categories to geographic features. For instance, each pixel on a raster map resulting from land cover classification is designated with a specific land cover type, implying complete membership in that category. The similarity model was developed within the context of fuzzy soil mapping to integrate fuzzy logic into the raster data model, aiming to provide a more realistic representation of geographic phenomena with inherent continuity and ambiguities (Zhu, 1997).

With the similarity model, a n-dimensional similarity vector is used to depict soil properties at each pixel location (i, j): $S_{ij} = ({S_{ij}}^1, {S_{ij}}^2, ..., {S_{ij}}^k, ..., {S_{ij}}^n)$, ${S_{ij}}^k$ where represents the similarity value or fuzzy membership of the soil at location (i, j) to a prototype of soil class k, and n is the total number of these soil classes. The prototypes, or central tendencies of properties associated with the mapped soil classes can be obtained using different methods. For example, they can be extracted from existing maps through a knowledge discovery approach (Qi et al., 2008). The knowledge of classifying spatial objects with prototype effects can also be acquired from domain experts in the form of fuzzy membership functions.

2.3 Fuzzy Membership Functions (FMFs) and Fuzzy Operators

FMFs quantify the degree to which an element belongs to a fuzzy set. These functions can assume various shapes. The triangular membership function is one of the simplest and most commonly used, featuring a triangular profile defined by the lower bound, the peak or center, and the upper bound. A trapezoidal membership function is similar, but with a flat plateau between its lower and upper bounds. These FMFs model linear changes in membership values. The Gaussian membership function is employed when membership uncertainty symmetrically distributes around a central value. A sigmoidal membership function is characterized by an S-shaped curve, both of which are frequently employed to simulate gradual transitions. Figure 1 illustrates FMFs used in fuzzy soil mapping.

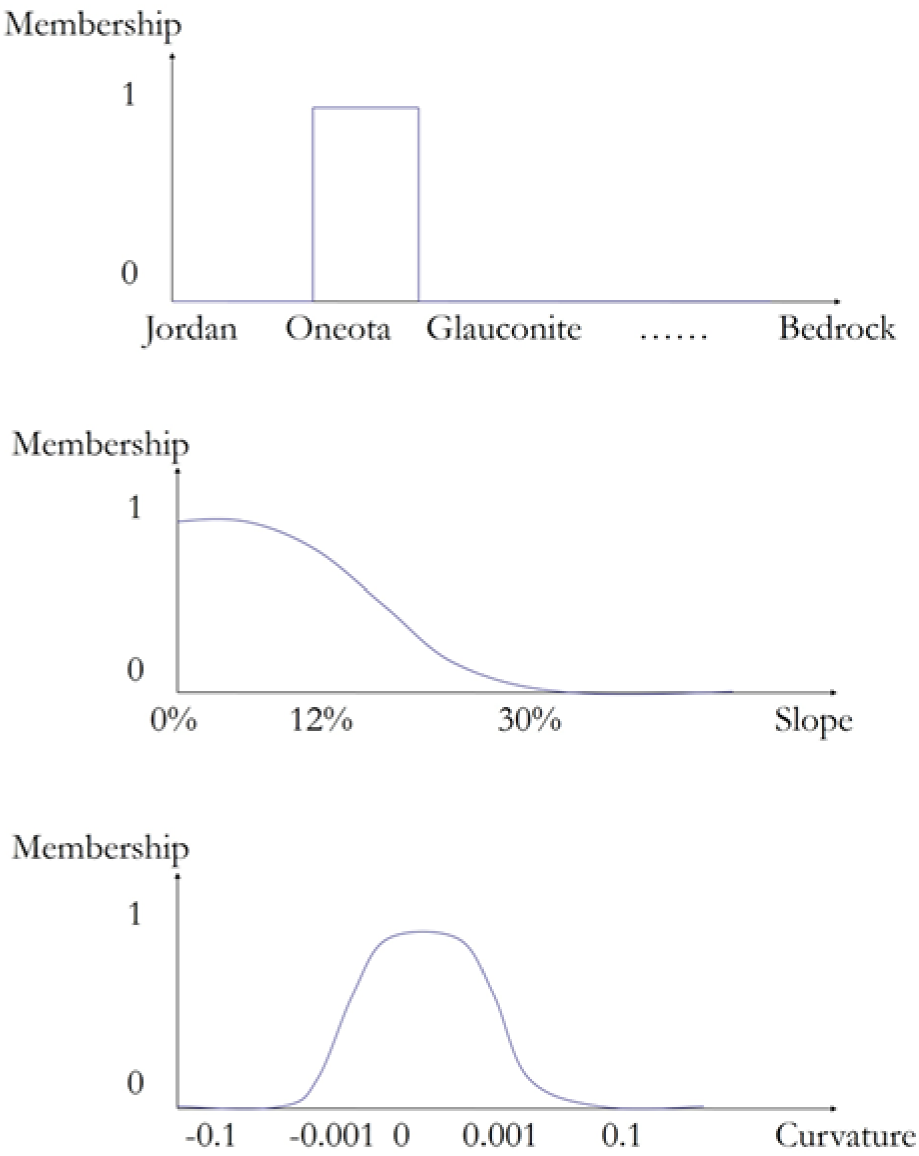

Figure 1. Fuzzy membership functions used in fuzzy soil mapping. Source: author.

FMFs depict the variation in membership degree based on specific factors or individual propositions. For instance, FMFs in figure 1 define the similarity degree to a soil type at a location based on bedrock geology, elevation, and slope steepness, respectively. Individual FMFs can be combined using fuzzy operators. Common operators include fuzzy intersection, union, complement, adapted from classical set theory. The fuzzy intersection (AND) operator merges two fuzzy sets A and B to form a fuzzy set with the degree to which elements belong simultaneously to both sets. This degree of membership in the intersection is determined by equation 1:

$$\mu_A \cap_B (x) = \min \{\mu_A(x), \mu_B(x)\}$$

Equation 1.

where $\mu_A(x)$ denotes the membership value of element x in fuzzy set A, and $\mu_B(x)$ is the membership value of element x in fuzzy set B.

Conversely, the fuzzy union (OR) operator combines fuzzy sets A and B with the degree to which elements belong to either set, by:

$$\mu_A \cup_B (x) = \max\{\mu_A(x), \mu_B(x)\}$$

Equation 2.

Fuzzy complement determines the extent to which an element x does not belong to fuzzy set A, expressed as

$$\mu \neg_A (x) = 1 - \mu_A(x)$$

Equation 3.

There are also several types of fuzzy implication operators, such as Mamdani, Larsen, and Lukasiewicz, which establish the relationship between antecedents and consequents in fuzzy rules.

2.4 Fuzzy Classification and Uncertainty Modeling

The uncertainty related to mapping and analysis errors can be modeled using stochastic methods. When geographic features exhibiting prototype effects are classified and mapped on area-class maps, the uncertainty related to class assignment, on the other hand, can be modeled with fuzzy memberships. There are two aspects of such uncertainty: the ignorance of individual instances' similarity to prototypes of other classes and the exaggeration of members' similarity to their own class prototypes. With geographic features represented as similarity vectors, the ignorance uncertainty can be approximated using an entropy measure:

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Equation 4.

where U_{ij} is the estimated ignorance uncertainty, S_{ij}^{k} is the similarity value of the instance at pixel (i, j) to class k, and N is the number of classes that the instance has similarity to . When the geographic feature at (i,j) has full membership to only one class, U_{ij} has the value 0 meaning no ignorance uncertainty is in question. The highest U_{ij} of the value of 1, on the other hand, indicates that the feature is evenly similar to all classes, and that assigning the instance to any one of the classes would involve the greatest degree of ignorance uncertainty.

The uncertainty associated with the exaggeration of members' similarity to their own class prototypes is inversely related to the saturation of its membership to the assigned class. If an instance bears complete similarity to the class prototype, there should be no exaggeration for the instance to be assigned to that class. And the lower the membership of the instance to the assigned class (to which the similarity is already the highest among all classes), the greater is the exaggeration. The exaggeration uncertainty can thus be approximated with:

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Equation 5.

where LaTeX parsing errored! is the estimated exaggeration uncertainty and

S_{ij}^{a} is the similarity value of the instance at pixel (i, j) to its assigned class (a).

The uncertainty measures can be calculated and mapped accompanying fuzzy classifications. Figure 2 shows uncertainty maps resulting from fuzzy soil classification.

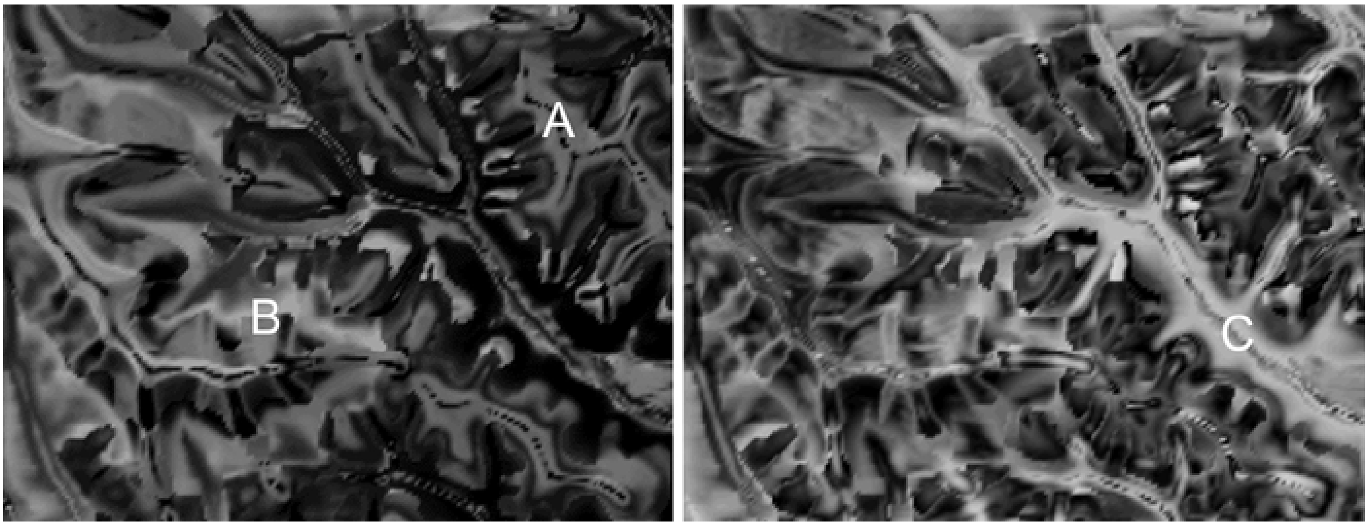

Figure 2. Distribution of the ignorance uncertainty (left) and exaggeration uncertainty (left) resulting from fuzzy soil classification with light tones indicating high uncertainty values, where ignorance uncertainty is high in transitional zones between soil types (A and B) and exaggeration uncertainty is high in an area © where the soil classification model is not able to predict soils accurately resulting in low similarities to any predefined soil classes. Source: author.

3. Example Areas of Applications

The applications of fuzzy modeling in GIS started in the 1990s (Fisher, 1992; Sui, 1994; Wang and Hall, 1996; Burrough and McDonnell, 1998). A diversity of applications emerged over the following three decades with several examples discussed below.

3.1 Predictive soil mapping

One of the primary applications of fuzzy models in GIS is fuzzy inference for decision-making. A fuzzy inference system consists of a knowledge base composed of FMFs and a set of fuzzy rules. Predictive soil mapping exemplifies a well-established fuzzy inference system. Zhu et al. (1996) introduced the soil-land inference model (SoLIM) for predictive soil mapping under fuzzy logic, which incorporated expert knowledge for fuzzy inference and employed similarity vectors to represent fuzzy soil class assignments. Within the SoLIM framework, different methodologies have been developed to acquire knowledge on the soil-landscape relationship in the form of FMFs. FMFs can be approximated from optimality curves directly acquired from soil experts with a personal construct-based approach (Zhu, 1999), derived from information extracted from conventional soil class maps with a knowledge discovery approach (Qi et al., 2008), or constructed from descriptive knowledge obtained through purposive field sampling (Zhu et al., 2010). With the FMFs, soils are inferred to bear various degrees of memberships to soil classes and continuous soil properties can be predicted from the memberships.

3.2 Risk modeling

Another primary application of fuzzy models in GIS is risk modeling, particularly in spatially explicit multi-criteria decision analysis (MCDA) where uncertainties and subjective assessments are significant. Fuzzy logic-based GIS models are commonly used to assess risks associated with natural hazards such as landslides, floods, and forest fires. Multiple layers of spatial data, representing topographical, hydrological, soil, land cover, and meteorological factors, can be evaluated using FMFs and integrated with fuzzy rules to assess susceptibility of landslides, floods, or forest fire.

In traditional MCDA, an analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions by breaking them down into hierarchical levels of criteria and alternatives. Fuzzy AHP extends the traditional AHP to handle uncertainty and vagueness in decision criteria and judgments by expressing judgments in linguistic terms (e.g., "somewhat important", "very important") and employing fuzzy aggregation techniques to compute overall priorities of alternatives. Examples of fuzzy aggregation operators include fuzzy weighted averaging (FWA), fuzzy ordered weighted averaging (FOWA) operators, and fuzzy integral operators (Mardani et al., 2018). Additionally, some applications in risk modeling utilize fuzzy Analytical Network Process (ANP), which extends fuzzy AHP by incorporating a network structure. This approach accommodates complex relationships and feedback loops among elements within and across hierarchy levels, enabling a dynamic representation of decision scenarios where interactions between criteria and alternatives nonlinearly influence outcomes (Abedi Gheshlaghi et al., 2019).

3.3 Suitability analysis and site selection

Another common application of fuzzy GIS models is suitability analysis and site selection. These models integrate multiple spatial data layers to identify optimal locations for conservation areas, green infrastructure, and sustainable development projects. For land suitability assessment, for example, fuzzy MCDA was performed with geostatistical interpolation models to evaluate nine criteria including terrain properties and soil features to identify potential sites suitable for agriculture lands in Central Anatolia Region (Özkan et al., 2020). In recent years, there has been a surge of applications using fuzzy logic for suitability analysis in site selection for renewable energy projects such as solar and wind farms.

4. Trends and future directions

A growing number of applications of fuzzy logic in GIS modeling utilize hybrid models or integrate fuzzy MCDA with advanced analytics such as AI/machine learning. Machine learning algorithms such as neural networks, support vector machines, and decision trees are employed to handle large datasets and derive insights that complement fuzzy MCDA evaluations. This integration enables the incorporation of complex patterns, nonlinear relationships, and large-scale data analysis into decision-making frameworks. For instance, Costache et al. (2022) developed hybrid approaches by integrating fuzzy AHP with Index of Entropy (IoE), Naïve Bayes, Multilayer Perceptron, and Deep Learning Neural Network models for predicting flood susceptibility. Similarly, fuzzy suitability analysis has witnessed increased integration with machine learning techniques and advanced spatial analytics (Elomiya et al., 2024). Future research aims to further enhance the integration of fuzzy logic with big data analytics to improve risk modeling and site selection processes.

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