[DM-05-047] Geographic Coordinate Systems

Abstract

Devised by Eratosthenes of Cyrene, geographic coordinate systems date back to the 3rd century BCE and have been used in mapping locations over the globe ever since. Latitude and longitude, or geographic coordinates, are the kernel of a geographic coordinate system. However, such a system is abstract unless anchored to the Earth in some way. The geodetic datum, or terrestrial reference frame as it is now called, comprises the anchor. Geographic information systems make extensive use of geographic coordinate systems to uniquely georeference spatial data. Here it is important to understand the uncertainty of a georeferenced ground position, given in latitude and longitude, as a function of the number of decimal places specified. Geocoding is a widely used application of geographic information systems that converts between street addresses and coordinates.

Keywords: coordinate systems, geoid, map projection, spatial reference systems

Author & citation

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Explanation

1. Definitions
2. What is a geographic coordinate system?
3. Geographic coordinates
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1. Definitions

- coordinate lines: a set of systematically dimensioned lines or curves along which coordinates are measured.
- coordinate system: a set of mathematical rules that assigns numbers, or coordinates, to uniquely describe the location of a point in space.
- curvilinear coordinate system: a coordinate system in which the coordinate lines may be curved.
- geodesic: an arc representing the shortest possible path between two points on a curved surface.
- geoid: the equipotential (or level) surface of the Earth's gravity field coinciding with the mean sea level of the oceans.
- graticule: an organized network of intersecting coordinate lines superimposed on a surface.
- great circle: the circle formed by the intersection of a sphere with a plane that passes

through the center of the sphere.
- meridian: a line of constant longitude formed by intersection of the sphere or ellipsoid with a plane containing the rotation axis.
- plumb line: a line everywhere tangent to the Earth's gravity vector.
- small circle: the circle formed by the intersection of the sphere or ellipsoid and a plane that is parallel to but not coincident with the equatorial plane.
- terrestrial reference system: a spatial reference system co-rotating with the Earth and with axes fixed to the solid Earth.
- terrestrial reference frame: a global set of three-dimensional points on the Earth's surface with known geocentric Cartesian coordinates and velocities that realizes a terrestrial reference system.

2. What is a geographic coordinate system?

A geographic coordinate system (GCS) is a graticule forming a curvilinear coordinate system overlaid on the surface of the Earth or extraterrestrial body and possessing specific properties. A GCS must:

- be fixed in some way to its underlying surface;
- have its coordinate lines intersect at the same angle everywhere;
- assign fixed numerical values to an arbitrarily selected origin;
- define the mathematical rules that assign numbers, or coordinates, along the coordinate lines.

A GCS provides unique location with respect to its underlying surface. As simple as it may sound, defining a GCS from first principles presents numerous complexities. We explore some of these complexities here.

All coordinate systems are abstractions. Abscissas (along x-axis) and ordinates (along y-axis) may be drawn on graph paper to represent the coordinates of a point in a two-dimensional, rectangular Cartesian coordinate system. Likewise, directional coordinates – such as longitude and latitude – may be “drawn” on the Earth’s surface to represent the East-West and North-South coordinates of a point in a geographic coordinate system. Both are abstractions or models of reality, there are no coordinate lines that we can see and follow on the surface of the Earth to guide us in the direction we wish to go. So, we define the coordinate system by convention (Teunissen et al., 2017).

Before defining a geographic coordinate system, we must decide on a reasonable geometric shape for the Earth. This will help to define the shape of the coordinate lines comprising the graticule. The Earth's shape is approximately spherical. However, Sir Isaac Newton argued in Principia Book III (1687), that because of Earth's rotatoin a more accurate shape is an ellipsoid of revoluion, also referred to as a spheroid, an oblate ellipsoid, or a biaxial ellipsoid (see Earth's Shape, Sea Level, and the Geoid). If a spherical shape for the Earth is chosen, then this suggests a set of orthogonally intersecting circles as a natural choice for the graticule. If the more accurate spheroidal shape is selected, then the graticule will comprise orthogonally intersecting circles and ellipses. In both cases, the center of the sphere or ellipsoid containing the graticule coincides with the Earth’s center (although this is not necessary).

Since the geographic coordinate system is a two-dimensional graticule overlaid on a three-dimensional Earth (or planetary body), we must anchor the graticule to the Earth's shape in a convenient way and then define a suitable starting point, or origin, for reckoning coordinates. A natural way to anchor the graticule is to make use of the Earth's rotation axis. The plane that intersects the rotation axis perpendicularly and is equidistant from the poles of the rotatoin axis is called the Equitorial plane (Figure 1). It divides the Earth into two parts called the Northern and Southern hemispheres. As the names imply, the Northern hemisphere lies "above" the Equatorial plane and the Southern hemisphere below the Equatorial plane. We are now in a position to construct the graticule in such a way as to render it a useful system for calculating real-valued coordinates that uniquely locate any point on the Earth – or, more precisely on the surface chosen to represent the shape of the Earth.

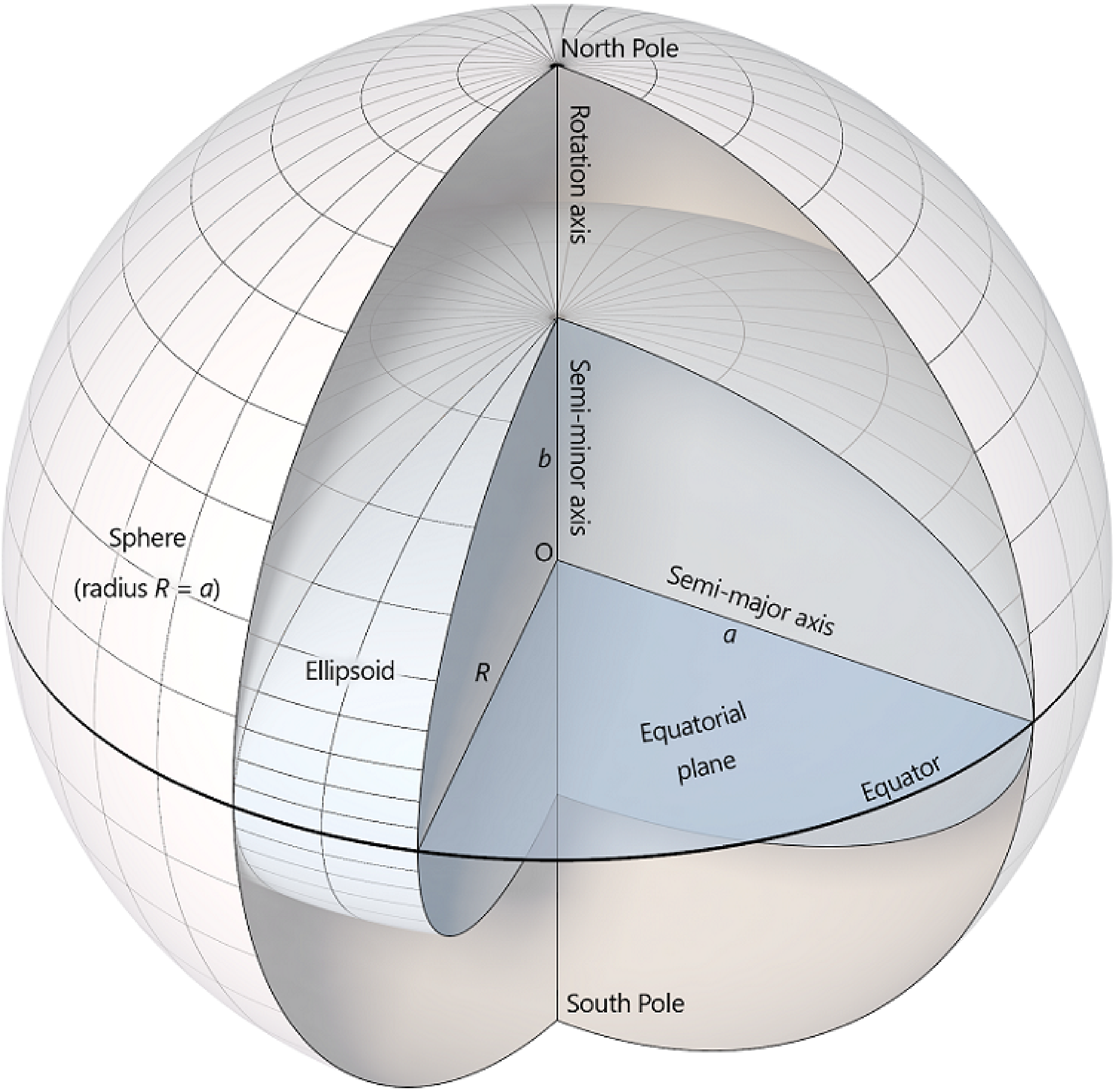

Figure 1. The Earth's rotation axis and equatorial plane. Spherical and ellipsoidal models of

the Earth are shown. The two models share the same rotation axis and coincide in the equatorial plane so that in both models the shape of the equator is a circle. Source: author.

Combined with a defined scale and orientation fixed to the Earth's surface, the resulting anchored graticule constitutes a geodetic datum, or in current geodetic terminology, a terrestrial reference frame (see Horizontal Datums). The geodetic datum, or reference frame, is a realization of a coordinate system suitable for calculating the positions and motions of points on Earth (Meyer, 2010). The datum comprises not only a set of stations on the Earth's surface with known coordinates but also any physical constants and models needed to compute the positions of other points (Kovalevsky et al., 1989).

A well-defined geographic coordinate system (GCS), or geographic coordinate reference system (GeoCRS), comprises the following elements: a shape specification such as a sphere or ellipsoid, a designated terrestrial reference frame or datum, a zero (or prime) meridian, and an angular unit of measure such as degrees or grads. In international standards such as ISO 19111 (ISO, 2019) and OGC Topic 2 (OGC, 2021), the shape is called a datum. The prime meridian defines the origin for calculating longitude – it is not unique and must be arbitrarily defined. Unlike the equatorial plane, which is normal to the Earth's rotation axis and equidistant from its poles, there are an infinite number of meridional planes containing the two poles and perpendicular to the equatorial plane. One of these meridional planes must be selected as the origin for measuring longitude. Since geographic coordinates – latitude and longitude – are angular measures, an angular unit must be specified.

3. Geographic Coordinates

Whether the shape chosen for the Earth is spherical or ellipsoidal, the mutually orthogonal coordinate lines of the graticule are called meridians and parallels, and the coordinates they define are called longitude and latitude, respectively. However, a distinction is made in latitude based on the chosen shape: geodetic or geographic latitude if ellipsoidal and spherical latitude and longitude if spherical. In practical usage a GCS, or in the language of international standards (ISO, 2019; OGC, 2021), a Coordinate Reference System (CRS), can refer to either a spherical or ellipsoidal (geographic) coordinate system.

3.1 Spherical Coordinates

Figure 2 illustrates spherical longitude and latitude. Let P be any point of interest. The angle θ between the line OP and the equatorial plane is called the spherical latitude of P where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Latitude is measured from 0° to 90° with positive sign north of the equatorial plane and negative sign south of the equatorial plane. North latitudes are often suffixed (or prefixed) with an N (e.g. 40°N) and South latitudes with an S (e.g. 40°S). A parallel of latitude is the locus of all points with the same latitude. With the additional property that the parallels are perpendicular to the rotation axis, all parallels of latitude except the equator are small circles.

As previously mentioned, since meridians are indistinguishable, one must be selected as the zero meridian (e.g. Greenwich Meridian). The angle λ, measured in the equatorial plane, between the zero meridian and the meridian plane of the point P is called the spherical longitude of P, where $0 \leq \lambda \leq 2\pi$. All meridians of longitude are great circles.

Conventionally (Torge, 2012; Meyer, 2010; ISO 6709, 2008), longitude is measured positive East from 0° to 360° , where 0° is the value assigned to the starting meridian. Longitude may also be calculated from 0° to ±180° with a positive sign east of the zero meridian and a negative sign west of the zero meridian. In this case, East longitude is often suffixed (or prefixed) with an E (e.g. 45°E) and West longitudes with a W (e.g. 45°W).

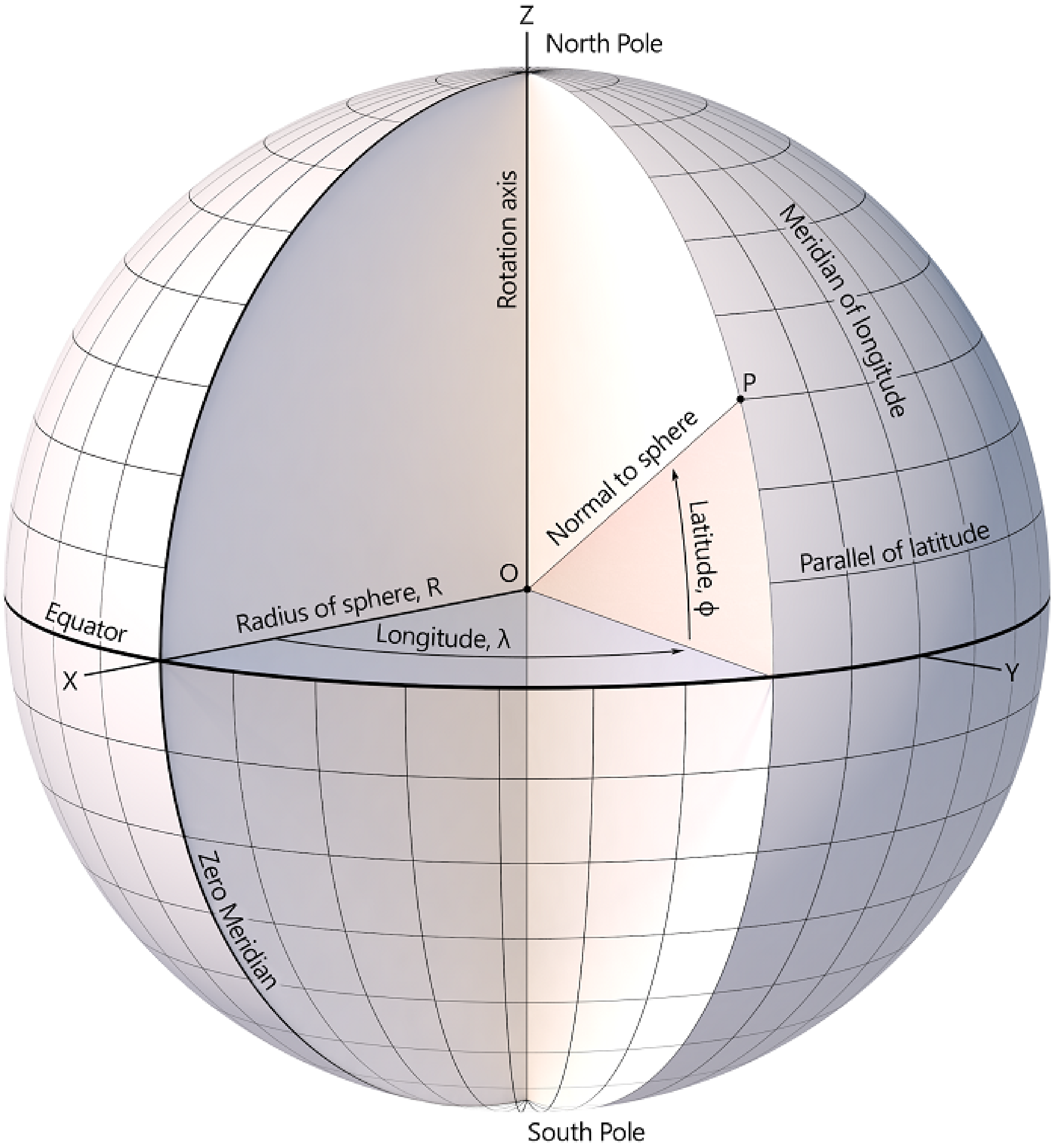

Figure 2. Spherical latitude and longitude. Source: author.

The position of the point P can also be expressed in Cartesian coordinates (X,Y,Z) as shown in Figure 2. The relationship between spherical and Cartesian coordinates is given by Equation 1:

$$X = R\cos\phi\cos\lambda$$
$$Y = R\cos\phi\sin\lambda$$
$$Z = R\sin\phi$$

(Equation 1)

where R is the radius of the sphere. The inverse relationship is given by Equation 2:

$$R = \sqrt{X^2 + Y^2 + Z^2}$$
$$\phi = \arcsin\frac{Z}{R} = \arctan\frac{Z}{\sqrt{X^2+Y^2}}$$
$$\lambda = \arctan\frac{Y}{X}$$

(Equation 2)

3.2 Ellipsoidal coordinates

As mentioned, a more accurate shape for the Earth than a sphere is an ellipsoid of revolution. An ellipsoid of revolution is obtained by rotating an ellipse about one of its principal axes. In the case of the Earth the rotation is about the minor axis and the resulting ellipsoid, termed the reference ellipsoid, is flattened at the poles and bulging at the equator. This is because the minor axis is shorter by about 21 km than the major axis. Defining parameters for the reference ellipsoid are usually given as its major axis a and either its minor axis b or flattening f. Ellipsoidal geometry along with ellipsoidal latitude and longitude are shown in Figure 3.

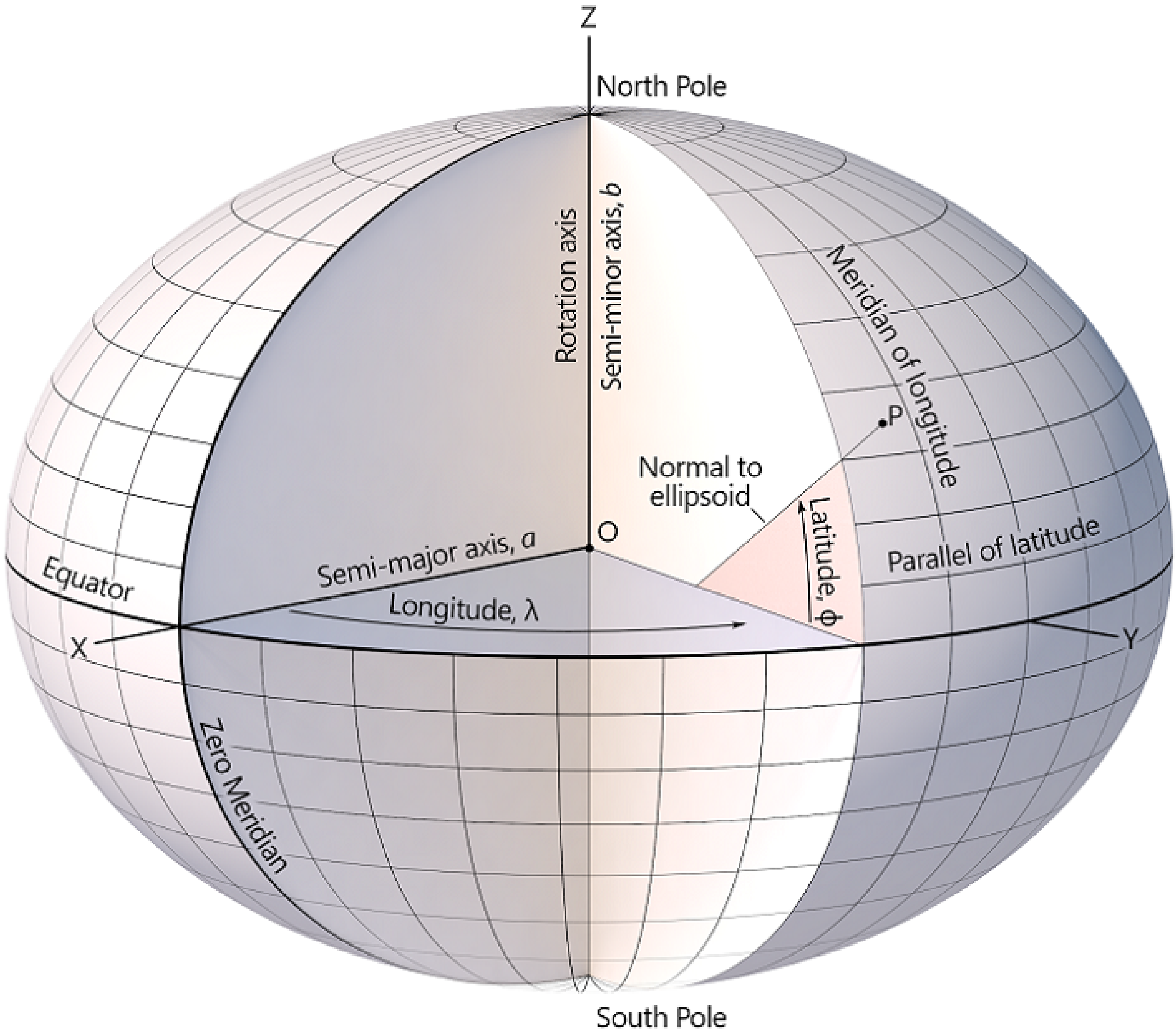

Figure 3. Ellipsoidal latitude and longitude. Source: author.

Again, let P be any point of interest. The angle Φ between the line CNP, which is normal to the ellipsoid, and the equatorial plane is called the ellipsoidal latitude of P, where $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$. Ellipsoidal latitude is measured in the same way as spherical latitude, although the ellipsoidal normal does not pass through the center of the ellipsoid. Ellipsoidal longitude is equivalent to spherical longitude. Ellipsoidal coordinates are also known as geographic or geodetic coordinates.

As with spherical coordinates, the position of the point P can also be expressed in Cartesian coordinates (X,Y,Z) as shown in Figure 3. The relationship between geographic and Cartesian coordinates is given by Torge and Muller (2012).

$$X = N \cos\phi \cos\lambda$$
$$Y = N \cos\phi \sin\lambda$$
$$Z = \left(1 - e^2\right) N \sin\phi$$

(Equation 3)

where e is the eccentricity of the ellipsoid given by:

$$e^2 = \frac{a^2 - b^2}{a^2},$$

(Equation 4)

and N is the radius of curvature in the prime vertical given by

$$N = \frac{a}{\left(1 - e^2 sin^2 \phi\right)^{1/2}}.$$

(Equation 5)

The inverse relationship is given by

$$\phi = \arctan \frac{Z}{(1-e^2)\sqrt{X^2+Y^2}}$$
$$\lambda = \arctan \frac{Y}{X}$$

(Equation 6)

Equations (1) - (6) apply to the case where the point P is on the surface of the sphere or ellipsoid, the more general case of P not being on the ellipsoid surface is addressed in the next section.

3.2.1 Geodetic coordinates

According to OGC (2021) and ISO (2019), a geographic coordinate system consists of a terrestrial reference frame, an ellipsoidal coordinate system and other associated elements (see Section 2). The ellipsoidal coordinates are geodetic latitude, geodetic longitude, and, in the three-dimensional case, ellipsoidal height. Ellipsoidal height, denoted by h, is the distance of a point from the reference ellipsoid measured along the ellipsoidal normal (see Figure 3). In this case, the relationship between geodetic and Cartesian coordinates is given by Torge and Muller (2012).

$$X = (N + h) \cos \phi \cos \lambda$$
$$Y = (N + h) \cos \phi \sin \lambda$$
$$Z = \left[\left(1 - e^2\right) N + h\right] \sin \phi$$

(Equation 7)

The literature is replete with methods for computing the inverse relationship, some iterative (e.g. Bowring, 1976; Borkowski, 1989; Fukushima, 2006) and others closed form (e.g. Hofmann-Wellenhof and Moritz, 2006; Vermeille, 2011; Turner and Elgohary, 2013). Iterative methods solve for ϕ and h using, for example Hofmann-Wellenhof and Moritz (2006).

$$\phi = \arctan\left[\frac{Z}{\sqrt{X^2+Y^2}}\left(1-e^2\frac{N}{N+h}\right)^{-1}\right]; \quad h = \frac{\sqrt{X^2+Y^2}}{\cos\phi} - N$$
$$\lambda = \arctan\frac{Y}{X}$$

(Equation 8)

In eqns. (8), begin by setting h=0 in the first equation and solving for latitude $\phi_{(1)}$. With $\phi_{(1)}$, compute an approximate value $N_{(1)}$ using eqn. (5). Then the second equation in (8) gives $h_{(1)}$, in turn yielding an updated value $\phi_{(2)}$ from the first equation. This iteration is repeated until $|\phi_{i+1} - \phi_i| < \delta$ and $|h_{i+1} - h_i| < \epsilon$, where δ and ϵ are specified tolerances.

4. Angular Measure

Taken along curved coordinate lines, latitude and longitude are measured in angular units. Four systems of units are in common use to quantify latitude and longitude: radians, decimal degrees, degrees-minutes-seconds, and gradians (grad or gon). Radian measure is used primarily in numerical computations involving latitude and longitude. To convert between degrees and radians, use:

$$1 radian = \frac{180^\circ}{\pi} \cong 57^\circ 295779513082320876...$$

(Equation 9)

Decimal degrees (DD) represent decimal fractions of a degree. They are commonly used in geographic information systems (GIS) and Global Navigation Satellite Systems (GNSS), such as GPS. Degrees-minutes-seconds (DMS) is a sexagesimal unit subdivision of a degree. In DMS units, one degree is divided into 60 minutes of arc (also called arcminutes), and one minute is divided into 60 seconds of arc (also called arc-seconds). In DMS units an angular value is written as, for example 39°10′40”. To convert DMS to DD, use:

$$DD = D + \frac{M}{60} + \frac{S}{3600}$$

(Equation 10)

For example, the decimal degrees representation of 34°03' 52"N, 117°11'47.23"W is

$$34 + \frac{3}{60} + \frac{36.52}{3600} = 34.0601\bar{4}$$

$$117 + \frac{11}{60} + \frac{47.23}{3600} = 117.196452\bar{7}$$

To convert DD to DMS, use:

$$D = \lfloor D\ D \rfloor\,,\ M = \lfloor 60\,|DD - D| \rfloor\,, S = 3600\,|DD - D| - 60M$$

where ⌊x⌋ is the floor function and|x| indicates absolute value. If DD is negative, ⌊x⌋ is replaced by the ceiling function ⌈x⌉. For example, the degrees-minutes-seconds representation of 34.06014, 117.1964527 is

$$\mathrm{D} = \lfloor 34.0601\bar{4} \rfloor = 34,\ \mathrm{M} = \lfloor 60|0.0601\bar{4}| \rfloor = 3, S = 3600|0.060144| - 60(3) = 36.51\bar{9}$$

$$= 34°03'36.52 = 34°03^{'}36.52"N$$

$$D = \lceil\text{-}117.196452\bar{7}\rceil = -117, M = \lceil 60|-0.196452\bar{7}| \rceil = -11, S = 3600|-0.196452\bar{7}| - 60(-11)$$
$$= -47.22\bar{9}$$

$$= -117°(-11)'(-47.23)" = -117°11'47.23" = 117°11'47.23"W$$

Gradians or gons (grad) employ a centesimal system of angular measurement. One grad or gon equals one hundredth of a right angle. This means there are 100 gradians in 90 degrees and 400 grads in a full 360° circle. Grads are commonly used in surveying, especially throughout Europe. To convert between grads, degrees and radians, use:

$$1\ grad\ = \frac{9}{10} degree = \frac{\pi}{200} radian$$

(Equation 12)

4.1 Linear Equivalents of Angular Measure

On a sphere of radius R, a simple formula relates angular measure to linear measure, or the length of the arc between two points,

$s = R\ \psi$
(Equation 13)

where ψ is the angle in radians subtended at the center of the sphere between the two points and s is the length of the arc between them in linear units (the same units as R).

Using this equation and letting $\psi = \Delta\lambda = \lambda_2 - \lambda_1$ be the longitude, we can construct Table 1 relating the longitude resolution to equivalent linear arc length. Moving away from the equator towards the pole, however, the linear arc length along a parallel decreases due to convergence of the meridians and approaches zero at the poles. In this case Eqn. (13) becomes

$s = R\ \Delta\lambda \cos\theta$
(Equation 14)

where θ is latitude. The relationship between latitude resolution and equivalent arc length on the sphere is the same as that of longitude on the sphere at the equator.

Table 1. Longitude resolution and equivalent arc length for a mean radius of the Earth, $R = 6{,}371{,}000\ m$.

Decimal Places	Decimal Degrees, $\Delta\lambda$	Equivalent DMS	Arc length at equator (m)	Arc length at 45°N (m)
0	1.0	1°0′0"	111194.927	78626.687
1	0.1	0°06′0"	11119.493	7862.669
2	0.01	0°0′36"	1111.949	786.267
3	0.001	0°0′3.6"	111.195	78.627
4	0.0001	0°0′0.36"	11.119	7.863
5	0.00001	0°0′0.036"	1.112	0.786
6	0.000001	0°0′0.0036"	0.111	0.079
7	0.0000001	0°0′0.00036"	0.011	0.008
8	0.00000001	0°0′0.000036"	0.001	0.0008
9	0.000000001	0°0′0.0000036"	0.0001	0.00008

The length of an arc of longitude on the ellipsoid depends on its eccentricity (related to the flattening) and the radius of a parallel of latitude and is given by Rapp (1991, Eqn. 3.122),

$$s_E = \frac{\Delta\lambda\,(a\cos\phi)}{\left(1 - e^2 sin^2\phi\right)^{1/2}}$$
(Equation 15)

Using this equation, we can construct Table 2 relating longitude resolution to equivalent linear arc length on the ellipsoid.

Table 2. Longitude resolution and equivalent arc length on the Geodetic Reference System 1980 (GRS80) ellipsoid: a$= 6{,}378{,}137\ m, b = 6{,}356{,}752.3141\ m.$

Decimal Places	Decimal Degrees, $\Delta\lambda$	Equivalent DMS	Arc length at equator (m)	Arc length at 45°N (m)
0	1.0	1°0′0"	111319.491	78846.835
1	0.1	0°06′0"	11131.949	7884.684
2	0.01	0°0′36"	1113.195	788.468
3	0.001	0°0′3.6"	111.320	78.847
4	0.0001	0°0′0.36"	11.132	7.885
5	0.00001	0°0′0.036"	1.113	0.789
6	0.000001	0°0′0.0036"	0.111	0.079
7	0.0000001	0°0′0.00036"	0.011	0.008
8	0.00000001	0°0′0.000036"	0.001	0.0008
9	0.000000001	0°0′0.0000036"	0.0001	0.0001

Fairly straightforward so far. Complexity increases when determining equivalent linear arc length in latitude on the ellipsoid. To do this we need to calculate the length of a portion of a meridian of longitude, or a meridional arc. Drawing on Rapp (1991, Eqn. 3.105) and Pallikaris et al (2009), the basic equation to calculate a meridional arc is

$$M_0^{\phi} = \int_0^{\phi} \frac{a(1-e^2)}{(1-e^2 sin^2\phi)^{3/2}} d\phi.$$

(Equation 16)

Equation (16) can be reduced to an elliptic integral of the second kind, which cannot be evaluated in closed form. Precise results can only be obtained using a binomial expansion of the denominator followed by integration term-by-term (Torge and Muller, 2012). Numerous methods have been derived to evaluate Eqn. (16) (e.g. (Vincenty, 1975; Bowring, 1983, 1985; Pearson, 1990). Vincenty (1975) has emerged as one of the most precise methods for calculating the arc of a meridian as well as accurate directions and geodesic distances on the ellipsoid. Vincenty (1975) is used to construct Table 3 relating latitude resolution to equivalent linear arc length on the ellipsoid.

Table 3. Latitude resolution and equivalent arc length on the Geodetic Reference System 1980 (GRS80) ellipsoid: a= $6{,}378{,}137\ m, b = 6{,}356{,}752.3141\ m.$

(Calculations performed using: https://geodesyapps.ga.gov.au/vincenty-inverse)

Decimal Places	Decimal Degrees, $\Delta\phi$	Equivalent DMS	Arc length at equator (m)	Arc length at 45°N (m)
0	1.0	1°0′0"	110574.389	111141.548
1	0.1	0°06′0"	11057.428	11113.275
2	0.01	0°0′36"	1105.743	1111.319
3	0.001	0°0′3.6"	110.574	111.132
4	0.0001	0°0′0.36"	11.057	11.113
5	0.00001	0°0′0.036"	1.106	1.111
6	0.000001	0°0′0.0036"	0.111	0.111
7	0.0000001	0°0′0.00036"	0.011	0.011
8	0.00000001	0°0′0.000036"	0.001	0.001
9	0.000000001	0°0′0.0000036"	0.0001	0.0001

From Tables 1 and 2, we can deduce that at the equator on the GRS80 reference ellipsoid: 1"λ ≈ $0.00027\overline{7}°$ ≈ 30.92 m, and on the sphere of radius R=6,371,000 m, the mean radius of the Earth, 1"λ ≈ 30.89 m. On the sphere 1"ϕ ≈ 30.89 m, the same as 1"λ at the equator. Table 3 shows that at the equator on the GRS80 reference ellipsoid: 1"ϕ ≈ 30.72 m at the equator and approximately 30.87 m at 45° N latitude.

Using the tables and equations in this section, one can deduce the uncertainty in latitude and longitude on the sphere and ellipsoid of a ground position specified in decimal degrees or decimal seconds to a given number of decimal places. For example, a GRS80 ellipsoidal longitude specified in decimal degrees to 5 decimal places represents a ground resolution of approximately 1.11 m at the equator and 0.79 m at 45° N latitude.

5. Auxiliary Latitudes

Various latitude definitions are used in special applications such as astronomy, geodesy, geophysics, and cartography. Astronomic latitude Φ and longitude Λ, together with the gravity potential W, form a natural three-dimensional coordinate set defined in the Earth's gravity field. The astronomic latitude of a point P is the angle between the direction of the plumb line at P and the equatorial plane.

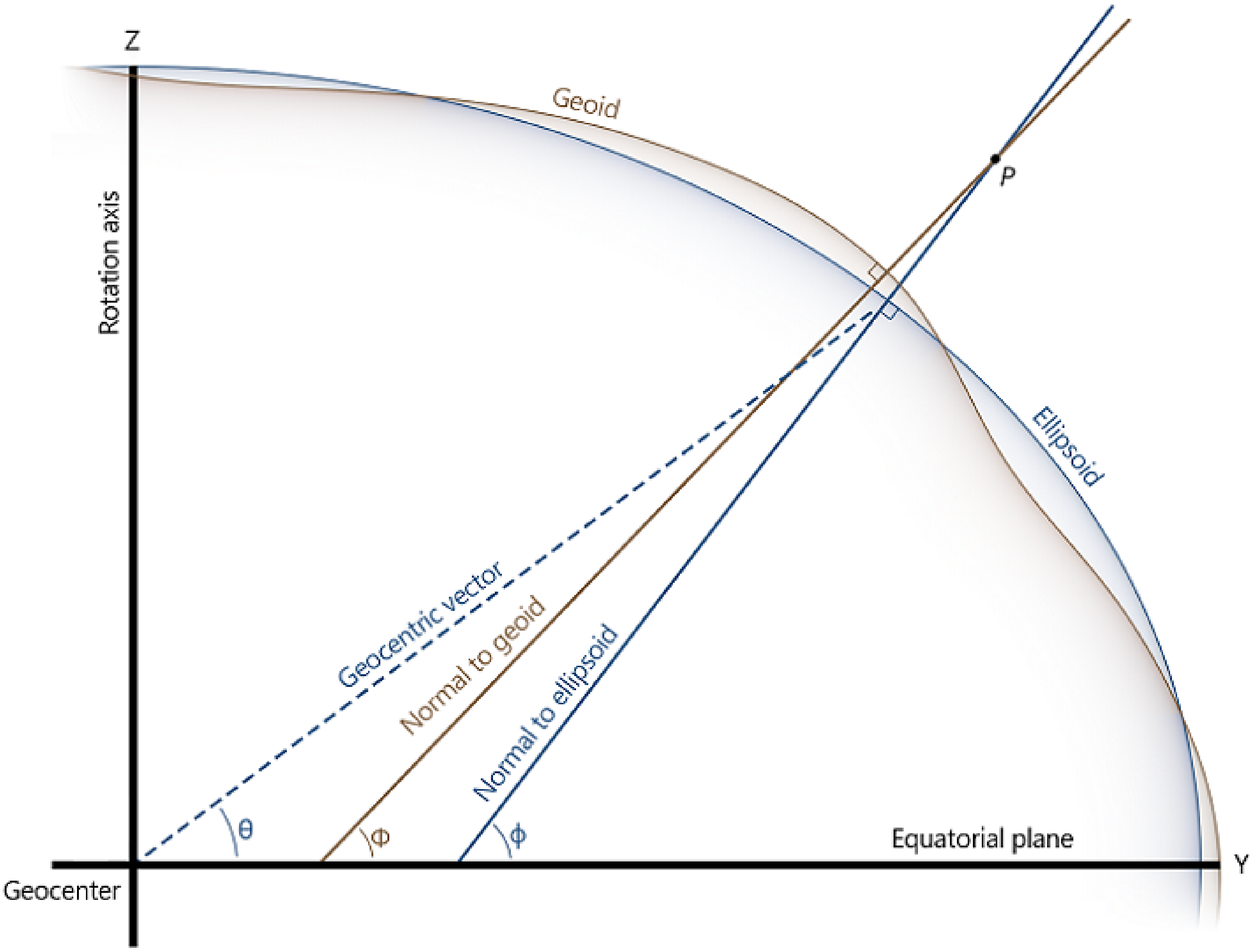

Figure 4. Geometric relationship between geodetic ϕ , astronomic Φ , and geocentric θ latitude. Source: author.

Since astronomic coordinates depend on the Earth's gravity field, which varies in magnitude and direction at different locations, so also astronomic positions vary as Earth's shape changes due to mass redistributions in the Earth (Meyer, 2010). Astronomic coordinates have been widely used to define local geodetic datums such as the North American Datum of 1927 (Schwarz, 1989).

Figure 4 illustrates the geometry of three types of latitude: geodetic latitude ϕ, astronomic latitude Φ, and geocentric latitude θ. Geodetic or ellipsoidal latitude has already been defined in Section 2.2. The angle between the equatorial plane and the radius from the geocenter to any point on the ellipsoidal normal is called geocentric latitude. Astronomical latitude is defined as the angle between the equatorial plane and any point on the plumb line.

Other auxiliary latitudes not illustrated in Figure 4 are: reduced or parametric latitude β, rectifying latitude μ, conformal latitude χ, authalic latitude ξ, and isometric latitude ψ. These latitudes arise in geodesy and geophysics as well as in map projections of the sphere or ellipsoid onto the plane, or in the calculation of geodesics on the ellipsoid. Detailed descriptions and derivations of these auxiliary latitudes can be found in, for example, Adams (1921), Snyder (1987), Osborne (2013), and Grafarend (2006). Formulas for these latitudes are given in Table 4 in terms of geodetic latitude ϕ.

Table 4. Formulas for some auxiliary latitudes.

[1]Geocentric latitude, θ	$\theta(\phi) = \arctan\left[\left(\frac{b}{a}\right)^2 \tan\phi\right] = \arctan[(1-e^2)\tan\phi]$
[2]Parametric latitude, β	$(\phi) = \arctan\left[\sqrt{1-e^2}\tan\phi\right]$
[3]Rectifying latitude, μ	$\mu(\phi) = \frac{\pi M_0^{\phi}}{2 M_0^{\pi/2}}$ where M_i^j is the meridional arc between $\phi_1 = i$ and $\phi_2 = j$ given by Eqn. (16);
[4]Conformal latitude, χ	$\chi(\phi) = 2\arctan\left[\frac{1+\sin\phi}{1-\sin\phi}\left(\frac{1-e\sin\phi}{1+e\sin\phi}\right)^e\right]^{1/2} - \frac{\pi}{2}$
[5]Authalic latitude, ξ	$\xi(\phi) = \arcsin\frac{q(\phi)}{q(\frac{\pi}{2})}$ $q(\phi) = \frac{(1-e^2)\sin\phi}{1-e^2\sin\phi} - \frac{1-e^2}{2e}\ln\left(\frac{1-e\sin\phi}{1+e\sin\phi}\right)$ $q\left(\frac{\pi}{2}\right) = \frac{1-e^2}{2e} ln\left(\frac{1-e}{1+e}\right)$
[6]Isometric latitude, ψ	$\psi(\phi) = \ln\left[\tan\left(\frac{\phi}{2}+\frac{\pi}{4}\right)\right] + \frac{e}{2}\ln\left[\frac{1-e\sin\phi}{1+e\sin\phi}\right]$

[1]The angle between the equatorial plane and the radius from the geocenter to any point on the ellipsoidal normal (see Figure 4).

[2]Defined by the radius drawn from the center of the ellipsoid to that point *Q* on the surrounding sphere (of radius *a*) which is the projection parallel to the Earth's axis of a point *P* on the ellipsoid at latitude ϕ (see Figure 4).

[3]Maps an ellipsoid (oblate spheroid) to a sphere while preserving the distances along the meridians (*Snyder*, 1987).

[4]Maps an ellipsoid (oblate spheroid) to a sphere preserving shapes and angles locally (*Snyder*, 1987).

[5]Maps an ellipsoid (oblate spheroid) to a sphere preserving surface area (*Snyder*, 1987).

[6]Directly proportional to the spacing of parallels of latitude from the equator on an ellipsoidal Mercator projection (*Snyder*, 1987).

6. Using a GCS

A GCS finds use in many contexts and activities: finding locations (coordinates) of places or geographic features on globes and maps, making maps or the science of cartography, studying and analyzing spatial relationships in a GIS, and many more. Most maps and GIS spatial databases reference a GCS.

6.1 Determining positions on a globe

A globe is a three-dimensional, extremely small-scale physical model of the Earth (or other celestial body). Globes are spherical and so slightly distort Earth’s actual shape. Globes are typically made with North at the top. Since the Earth's circumference is about 40 million meters globes with a circumference of one-meter model the Earth at a scale of 1:40,000,000. Globes with a diameter of one foot, yielding a circumference of 3.14 feet, model the Earth at a scale of 1:42,000,000.

Parallels of latitude and meridians of longitude appear on the globe only at wide intervals, so as not to obscure the geographic features. Angular measure is used in describing the location of places on the globe. Because of the small scale of globes, the separation of parallels of latitude is large, typically 10 or 15 degrees, sometimes larger, with a similar separation for the meridians of longitude. The intersection of a parallel of latitude and a meridian of longitude uniquely defines the location of a point on the globe (Figure 5).

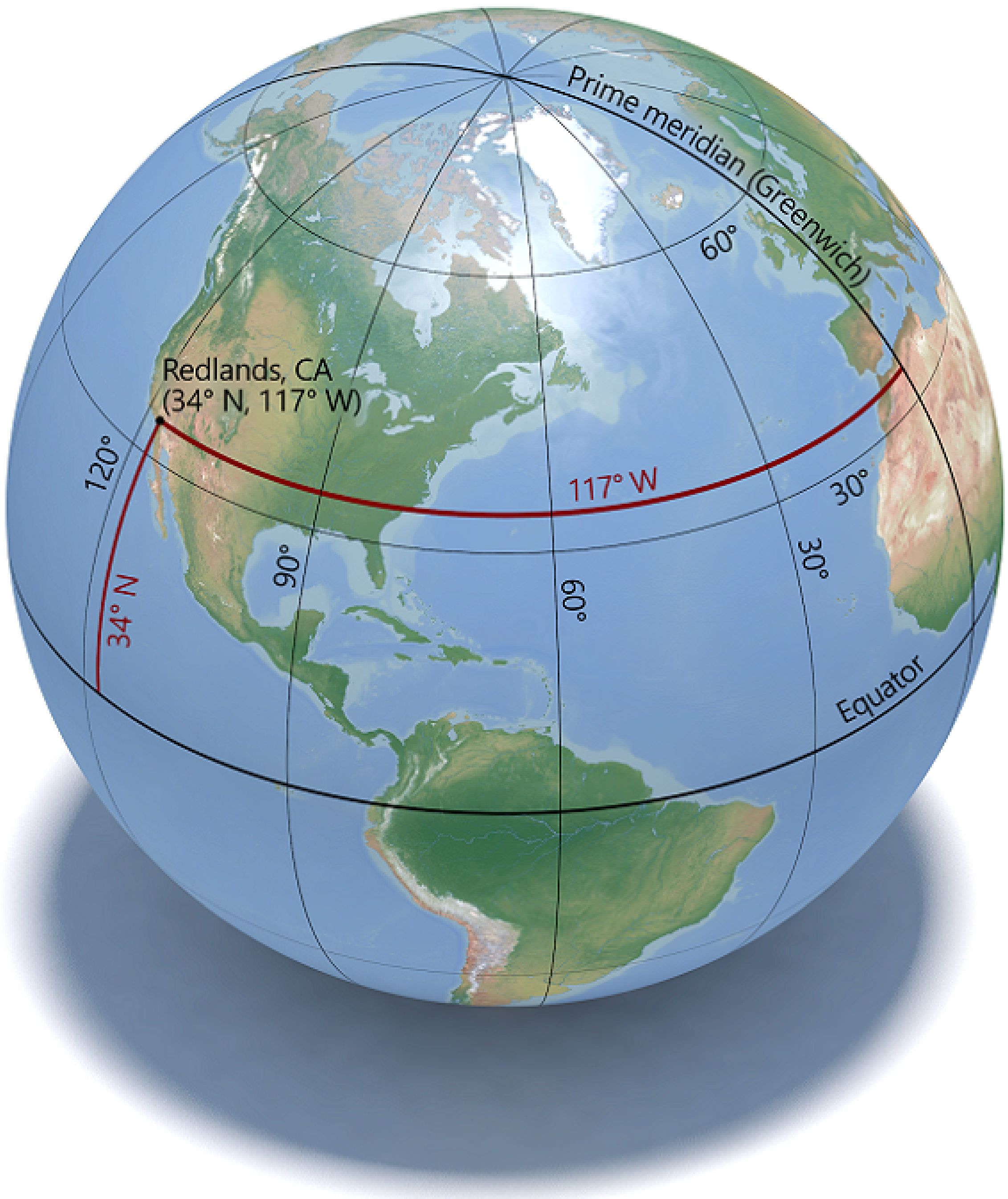

Figure 5. Locations on a globe are uniquely determined by the point where the longitude and latitude lines intersect. For example, Redlands, California is located at 34° N latitude (north of the Equator) and 117° W longitude (west of the Prime Meridian). Source: author.

6.2 Calculating coordinates on a map

When the graticule appears on a map we can usually calculate or plot the geographic coordinates of any location on the map. The larger the map scale the more accurate the calculation can be performed. Graticules on a map may appear as tick marks on the map edge or as crosses in the interior of the map. Figure 6 shows examples of the graticule on different types of maps: topographic, hydrographic, and aeronautical. Notice the graticule is shown as coordinate grid ticks in the collar of the map though in different ways on each of the maps.

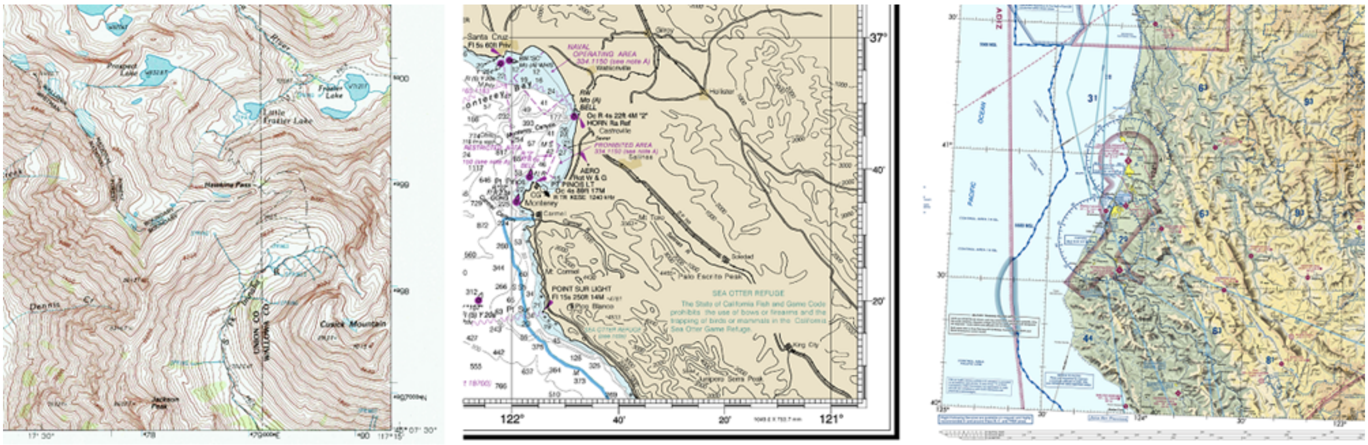

Figure 6. (A, left) Graticule on a U.S. Geological Survey topographic map, (B, center) Graticule on a U.S. Office of Coast Survey nautical chart, and (C, right) Graticule on a U.S. Federal Aviation Administration aeronautical chart. Source: author.

The angular resolution represented by graticule tick marks depends on map scale. The nautical chart snippet shown in Figure 6 (B) has a graticule resolution of one-minute of arc. Here, positions of locations on the chart may be determined directly to the nearest 30”, or slightly better. Similarly, for the aeronautical chart snippet shown in Figure 6 (C). Position determination on the topographic map in Figure 6 (A) requires more effort. We give a general method for this case but can be applied to any map or chart displaying a graticule In Figure 7, we desire to calculate the geographic coordinates of the point P using the graticule printed on the map. The procedure is as follows:

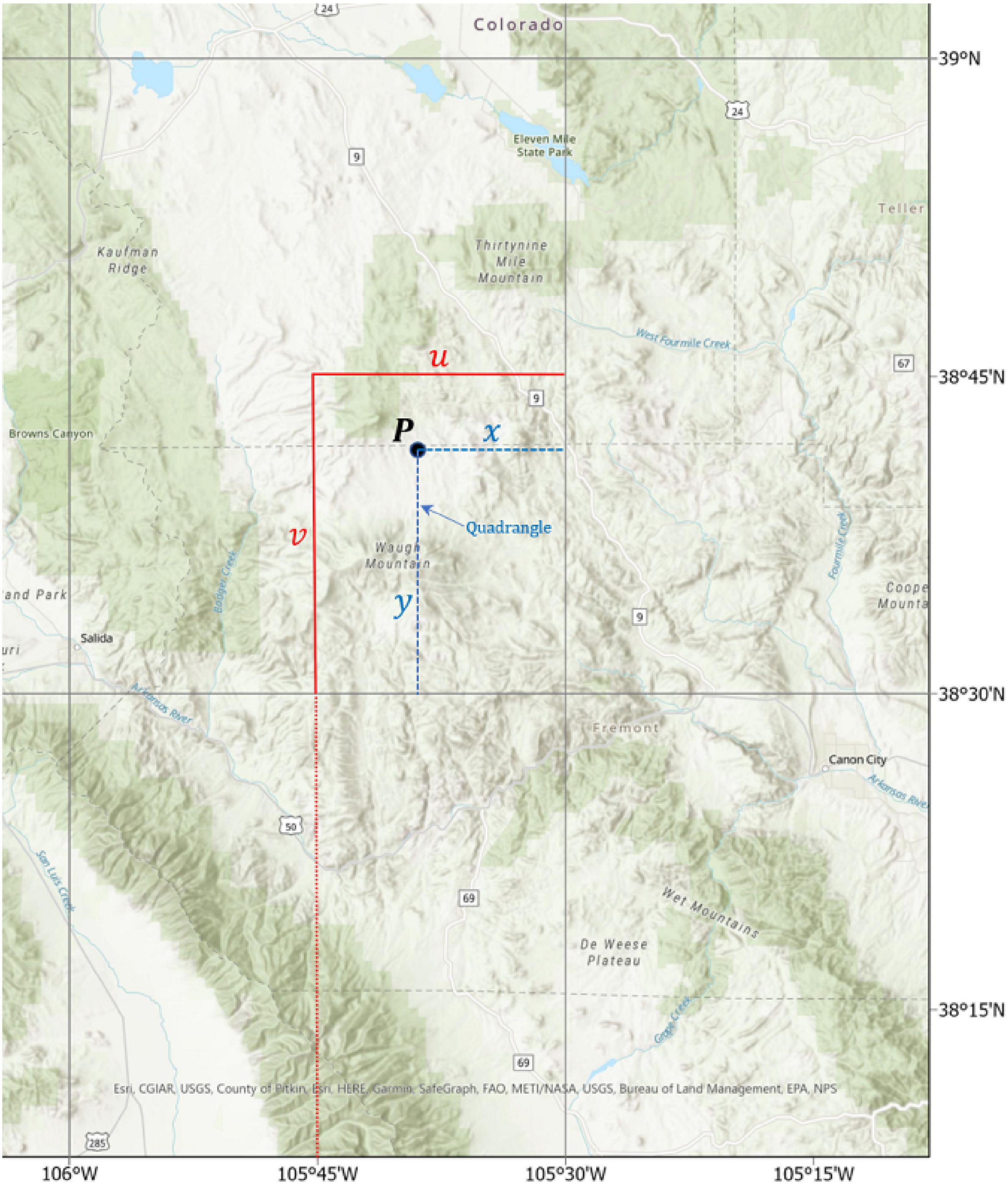

Figure 7. Calculation of geographic coordinates of a location P using the graticule printed on the map. Source: author.

1. Mark the location of the point P on the map.
2. Draw the smallest possible quadrangle with the point P at one vertex. The line x should be drawn parallel to the closest parallel of latitude to P with P at one vertex and the opposite vertex meeting at the nearest meridian of longitude or tick mark printed on the map. The line y should be drawn parallel to the closest meridian of

longitude to P with P at one vertex and the opposite vertex meeting at the nearest parallel of latitude or tick mark printed on the map. Thus, P should form one vertex of the quadrangle connecting the four nearest intersections of parallels and meridians, or their representative tick marks.

3. If needed, draw the quadrangle connecting the four nearest intersections of parallels and meridians, or their representative tick marks surrounding P. This defines the solid lines u and v in Figure 7.
4. Using a ruler, preferably one graduated in millimeters, measure the lengths x,y,u, and v. Let the interval between latitude and longitude tick marks of the quadrangle surrounding P be $\Delta\phi$ and $\Delta\lambda$, respectively (not shown in Figure 7).

Use linear interpolation to calculate the latitude offset $d\phi$ from the closest parallel of latitude,

$$\frac{\Delta\phi}{v} = \frac{d\phi}{y} \quad \text{or} \quad d\phi = \frac{y}{v}\Delta\phi$$

(Equation 17)

Similarly, use linear interpolation to calculate the longitude offset $d\lambda$ of P from the closest meridian of longitude,

$$\frac{\Delta\lambda}{u} = \frac{d\lambda}{x} \quad \text{or} \quad d\lambda = \frac{u}{x}\Delta\lambda$$

(Equation 18)

6. Apply the respective offsets to the known values of the tick marks to obtain the desired latitude and longitude of P.

For example, in Figure 7, let x = 24 mm, y = 40 mm, u = 41 mm, and v = 52 mm. From the map graticule, $\phi_{Tick} = 38°30',\ \Delta\phi = 15',\ \lambda_{Tick} = 105°30', \text{and } \Delta\lambda = 15'$. Then, the latitude offset is,

$$d\phi = \frac{40}{52}(15) = 11.54' = 11°32"$$

The final geographic coordinates of P then are,

$$\phi_p = \phi_{Tick} + d\phi \;=\; 38°30' + 0°11'32" \;=\; 38°41'32"\,N$$

7. Geocoding

Geocoding is the process of taking an address or place name and converting it to geographic coordinates by comparing the descriptive information to a set of reference data. Reference data may consist of rooftop address points, street network data, parcel boundaries, postal codes, geographic features, populated places, or some combination of them (Zandbergen, 2008). Reverse geocoding converts geographic coordinates to an address or place name. Geocoding relies on a computer representation of addresses, streets and administrative boundaries and is typically performed in a GIS.

The geocoding process comprises separating addresses into pieces, such as number, street name, and street type and normalizing them based on a set of rules defined in geocoding algorithm. A search is made for the normalized address in the reference data, candidate locations are returned, compared, and scored, and a list of best matches provided. The candidate with the highest score is used for generating geographic coordinates for the location of the matched address. The core and most complex part of the geocoding process is the matching algorithm that picks the best feature in the reference dataset. The output geographic location may be derived from a polygon centroid or a linear interpolation (Figure 8) of the location of the street number within an address range (Goldberg et al., 2007; Zandbergen, 2008).

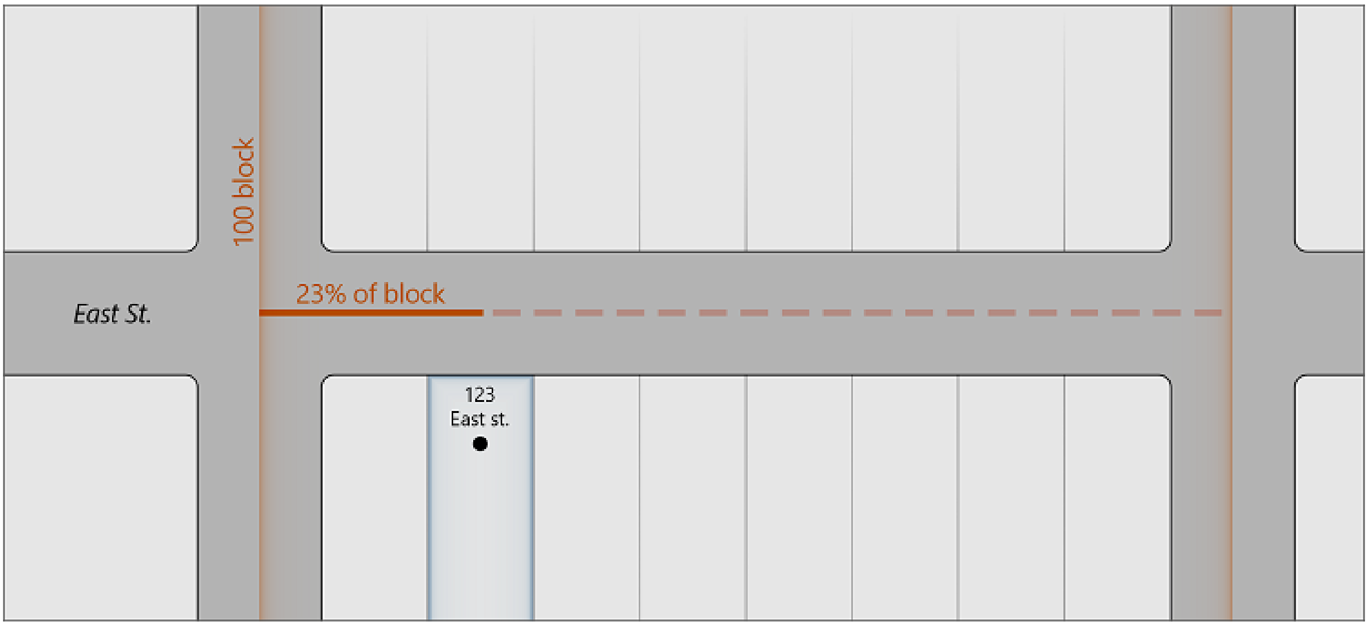

Figure 8. Geocoding with street centerline data outputs approximate geographic coordinates of the matched address Typically, an offset is applied to the position to ensure it is not located in the center of the street. Geocoding with centroid data usually provides a more accurate location of a building or parcel than using street centerlines. Source: author.

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